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Estimating the Efficacy of Mass Rescue Operations in Ocean Areas with Vehicle Routing Models and Heuristics

Doutoramento em Estatística e Investigação Operacional
Especialidade de Otimização

Rui Pedro Gonçalves de Deus

Tese orientada pelo:
Prof. Doutor Luís Eduardo Neves Gouveia

Documento especialmente elaborado para a obtenção do grau de doutor

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2018

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Epigraph

"Basically because it's a big problem for our members. The IMO define a mass rescue operation (MRO) as being characterised by the need for immediate response to large numbers of persons in distress such that the capabilities normally available to the SAR authorities are inadequate, which is an IMO way of saying it's almost too big to handle. Well, it's certainly big and it's certainly difficult and it's certainly pretty scary but we believe that properly prepared, it isn't too big to handle and it's that preparation which is the key."

David Jardine-Smith, Secretary, IMRF (answering why MRO are such a big priority for the IMRF)

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Abstract

Mass rescue operations (MRO) in maritime areas, particularly in ocean areas, are a major concern for the authorities responsible for conducting search and rescue (SAR) activities. A mass rescue operation can be defined as a search and rescue activity characterized by the need for immediate assistance to a large number of persons in distress, such that the capabilities normally available to search and rescue are inadequate. In this dissertation we deal with a mass rescue operation within ocean areas and we consider the problem of rescuing a set of survivors following a maritime incident (cruise ship, oil platform, ditched airplane) that are drifting in time. The recovery of survivors is performed by nearby ships and helicopters. We also consider the possibility of ships capable of refuelling helicopters while hovering which can extend the range to which survivors can be rescued. A linear binary integer formulation is presented along with an application that allows users to build instances of the problem. The formulation considers a discretization of time within a certain time step in order to assess the possibility of travelling along different locations. The problem considered in this work can be perceived as an extension of the generalized vehicle routing problem (GVRP) with a profit stance since we may not be able to recover all of the survivors. We also present a look ahead approach, based on the pilot method, to the problem along with some optimal results using state of the art Mixed-integer linear programming solvers.

Finally, the efficacy of the solution from the GVRP is estimated for a set of scenarios that combine incident severity, location, traffic density for nearby ships and SAR assets availability and location. Using traffic density maps and the estimated MRO efficacy, one can produce a combined vulnerability map to ascertain the quality of response to each scenario.

Key words: mass rescue operations, SAR system efficacy, vehicle routing problem, pilot method

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Resumo

(Abstract in portuguese)

Uma operação de salvamento em larga escala caracteriza-se pela necessidade de prestar auxílio imediato a um elevado número de pessoas que, nesse momento, se encontram em risco de vida em circunstâncias em que os meios e capacidades ao nível do sistema de busca e salvamento não são os adequados para garantir uma resposta imediata. Este tipo de operação ocorre com uma frequência inferior, quando comparado com as típicas operações de busca e salvamento, no entanto, os incidentes que requerem estas operações acarretam consequências de elevado valor em termos de vidas humanas e prejuízos materiais. Operações desta natureza poderão envolver centenas ou mesmo milhares de pessoas que necessitam de socorro em ambientes por vezes longínquos e adversos à condição humana. Inundações, terramotos, ataques terroristas, acidentes em plataformas petrolíferas situadas em zonas costeiras ou a colisão de um navio de passageiros de grande porte são exemplos, que, pela sua magnitude, podem requerer o empenhamento de meios de salvamento necessários para realizar uma operação de salvamento em larga escala. A existência de planos de contingência e de uma capacidade de resposta eficaz constituem fatores-chave para evitar as consequências já referidas.

Do vasto leque de operações de salvamento em larga escala existente, o presente trabalho particulariza aquelas que ocorrem em ambiente marítimo, ou seja, no mar. Os incidentes que poderão requerer uma operação de salvamento marítimo em larga escala são múltiplos e apresentam diferentes características. Em geral, o denominador comum neste tipo de incidentes reside no facto de existir um número elevado de pessoas que precisam de ser socorridas sem que haja forma de fazer chegar meios de salvamento de forma imediata ao local do incidente e com a capacidade de salvamento necessária. Por exemplo, um acidente numa plataforma petrolífera, onde trabalham cerca de 100 pessoas, poderá constituir um incidente com um número elevado de vítimas mortais, caso não seja possível atuar de forma imediata. Neste tipo de plataformas, que, de uma forma geral, se situam em zonas costeiras, e por conseguinte, estão relativamente próximas de meios de socorro, aquando da chegada ao local do incidente dos primeiros meios de salvamento, os sobreviventes poderão encontrar-se já a flutuar na água, ou resguardados em jangadas ou embarcações de salvamento pertencentes à plataforma. As condições meteorológicas no local e o tempo que os sobreviventes permanecem na água após a ocorrência de um incidente constituem fatores críticos no planeamento de uma operação desta natureza. As condições meteorológicas são

um fator crítico, quer para a sobrevivência dos sinistrados após o incidente, quer para a sua recuperação. Se as condições atmosféricas e oceanográficas forem adversas, a recuperação dos sobreviventes que se encontram na água poderá não ser exequível ou demorar mais tempo que o normal. O tempo que os sobreviventes passam na água ou em jangadas até que sejam recuperados por meios de salvamento poderá variar entre poucos minutos e várias horas. No caso de o incidente ocorrer em águas oceânicas, tanto os sobreviventes na água como aqueles que se encontram resguardados em jangadas ou embarcações de salvamento irão estar sujeitos ao efeito da deriva marítima. Em águas oceânicas, o movimento de objetos na superfície da água é influenciado principalmente por duas componentes da deriva marítima: o abatimento e as correntes de vento. Caso a chegada dos meios de busca e salvamento ao local do incidente seja demorada, a ação destas forças sobre os sobreviventes, jangadas e embarcações irá dispersá-los e tornar a sua recuperação mais morosa e difícil.

Este tipo de operações viu a sua importância ser reconhecida com a realização de cruzeiros turísticos a zonas do Ártico. Estas zonas não possuem infraestruturas de salvamento nas suas proximidades com capacidade adequada para responder a um possível incidente que requeira uma operação de salvamento em larga escala. Este tipo de atividade suscitou a preocupação do Comité de Segurança Marítima da International Maritime Organization (IMO) que promoveu a discussão do risco associado a navios de cruzeiro a operar em áreas remotas entre os seus Estados membros e organizações internacionais com assento neste comité. Fruto desta discussão, foi reconhecido que o conceito de "áreas remotas" não se encontra circunscrito a zonas longíquas como o Ártico e que estas áreas poderão existir nas regiões de busca e salvamento marítimo dos Estados costeiros em zonas bem mais próximas de infraestruturas de salvamento do que se pensava até então. Para além desta situação, a ameaça de ataques terroristas a navios de cruzeiro contribuiu também para aumentar a preocupação das entidades governamentais dos Estados costeiros em relação às operações de salvamento em larga escala.

Neste trabalho pretende-se estimar a eficácia de uma operação de salvamento em larga escala através de modelos de otimização combinatória para problemas de roteamento de veículos. Para este efeito, é formulado o problema de salvamento marítimo em larga escala, designado por problema MMRO, que consiste numa variante do problema de roteamento de veículos generalizado onde se pretende visitar um conjunto de clientes que correspondem a objetos SAR (pessoas na água, jangadas salva-vidas e embarcações à deriva) através de um conjunto de veículos heterogéneos que se deslocam com diferentes velocidades. Este problema tem a particularidade de considerar a deriva dos objetos ao longo do tempo por estes estarem sujeitos ao efeito da deriva marítima. Para além da modelação do problema através de modelos combinatórios, pretende-se avaliar de que forma o conhecimento dos tempos de sobrevivência por parte do sistema SAR tem impacto na eficácia das operações de salvamento. Este tipo de informação é utilizado num critério para prioritizar os objectos a serem recuperados por um determinado veículo. Para este efeito, são testados diferentes variantes de heurísticas para o problema MMRO, que incorporam diferentes critérios de prioridade. Uma das questões que se procura responder prende-se com o conhecimento do estado de saúde das pessoas que estão na água ou em jangadas salva-vidas por parte de quem está a a coordenar os veículos, quer estes sejam aéreos ou navais, e se essa informação pode ser usada para uma coordenação das ações de recuperação de sobreviventes mais eficiente. Nos procedimentos de recuperação *standard*, um veículo utiliza o critério da distância para decidir qual o objecto que irá recuperar. No caso de existir mais do que um veículo a operar, então o critério é baseado no tempo de viagem até chegar ao objecto. Estas variantes são incorporadas num procedimento de *look ahead*, designado por método piloto, de forma a resolver instâncias do problema MMRO que os métodos exactos não são capazes de resolver.

De forma a permitir a criação de instâncias que representem um incidente envolvendo vários objetos SAR, é desenvolvido um protótipo em MATLAB que disponibiliza diversas funcionalidades de um sistema de informação geográfico para colocação e manuseamento de objetos num mapa e cálculo de distâncias. Este protótipo permite ainda ao utilizador selecionar vários tipos de heurísticas, entre as quais diversas variantes do método piloto e analisar a sua eficácia e outras características associadas à solução (tempo médio que cada objeto permaneceu na água até ser recuperado, número de milhas percorridas por cada veículo, etc). Este protótipo afigura-se como um instrumento de estudo para avaliar a resposta de um sistema SAR a um incidente localizado numa área oceânica e permite efetuar uma análise de sensibilidade em função da disponibilidade de meios de salvamento.

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(Acknowledgments in portuguese)

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Este trabalho é para ti, Pedro! Que te sirva de exemplo e de motivação para os desafios que irás abraçar!

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Abbreviations and Acronyms

<u>A</u>

AIS	Automatic Identification System
AISINTEL	Ferramenta de análise e visualização de dados AIS e MONICAP
AGV	Automated Guided Vehicles
AM	*Ante merīdiem* (before midday)
ASWORG	Antisubmarine Warfare Operations Research Group

<u>B</u>

BSM	Busca e Salvamento Marítimo (search and rescue in portuguese)
BWTSP	Black-and-white traveling salesman problem

<u>C</u>

C2	Command and Control
CINAV	Portuguese Naval Research Center
CLIA	Cruise Lines International Association
CODU	Centro de Orientação de Doentes Urgentes
COI	Contact Of Interest
COMAR	Centro de Operações Marítimas
COMNAV	Naval Command
COMSAR	Sub-Committee on Radio Communications and Search and Rescue
ConFL	Connected Facility Location problem
CPU	Central Processing Unit
CSW	Critical Software
CTSP	Capacitated traveling salesman problem
CVRP	Capacitated vehicle routing problem
CGWRP	Capacitated General Windy Routing Problem

<u>D</u>

DAGI	Direção de Análise e Gestão da Informação
DGRM	Direção Geral de Recursos Naturais, Segurança e Serviços Marítimos
DOM	Disaster Operations Management
DSS	Decision support system
DTG	Date Time Group

<u>E</u>

EADS	European Aeronautic Defence and Space Company
EEZ	Economic Exclusive Zone
EFF	Efficacy
EMSA	European Maritime Safety Agency
EPIRB	Emergency Position-Indicating Radio Beacon
ETA	Expected Time Arrival

F

FIR	Flight Information Region

G

GAMA	Accident Investigation Bureau Maritime and Aeronautical Meteorology Authority
GIS	Geographic information system
GDH	Grupo Data Hora
GmTSP	Generalized multiple traveling salesman problem
GPS	Global Positioning System
GRIB	GRIdded Binary
GTD	Global terrorism database
GTSP	Generalized traveling salesman problem
GVRP	Generalized vehicle routing problem
GVRPTW	Generalized vehicle routing problem with time windows

H

HIFR	Helicopter in-flight refuelling
HMSTP	hop-constrained minimum spanning tree problem

I

IALA	International Association of Marine Aids to Navigation and Lighthouse Authorities
IAMSAR	International Aeronautical Maritime Search and Rescue manual
IED	Improvised explosive device
ICAO	International Civil Aviation Organization
ILP	Integer linear programming
IMO	International Maritime Organization
IMRF	International Maritime Rescue Federation
IT	Information Technology
ITU	International Telecommunication Union

K

Kts	Knots (nautical miles per hour)

L

LLA	Lives lost after notification
LLB	Lives lost before notification
LNS	Large neighbourhood search
LS	Lives saved
LUF	Lives unaccounted for
LW	Leeway

M

MATLAB	MATrix LABoratory
MCARPTC	Mixed capacitated routing problem with turn constraints
MEDEVAC	Medical evacuation
MDGVRP	Multiple depot generalized vehicle routing problem
MDOVRP	Multi-Depot Open Vehicle Routing Problem
MDT	MMRO Design Tool
MDTSP	Multiple-depot Traveling Salesman Problem
MDVRP	Multiple-depot Vehicle Routing Problem
MIT	Italian Ministry of Infrastructure
MMRO	Maritime Mass Rescue Operation
MMSI	Maritime Mobile Service Identity
MoD	Ministry of Defense
MONICAP	Monitorização Contínua das Atividades da Pesca
MRCC	Maritime Rescue Co-ordinator Centre
MRO	Mass Rescue Operation
MROSS	Mass Rescue Operations Scoping Study
MRSC	Maritime Rescue Sub-Center
MSP	Maritime spatial planning

<u>N</u>

Nm	Nautical Miles
NNH	Nearest Neighbour Heuristic
NTSB	National Transportation Safety Board

<u>O</u>

OP	Orienteering problem
OPV	Ocean Patrol Vessel
OSC	On-scene co-ordinator

<u>P</u>

PCTSP	Prize Collecting Traveling Salesman Problem
PCVRP	Prize Collecting Vehicle Routing Problem
PIW	Person in the water
PL	Value of Property Lost
PLP	Value of Property Loss Prevented
PTP	Profitable tour problem
PUF	Property unaccounted for

<u>R</u>

RADAR	RAdio Detection And Ranging
RCC	Rescue Coordinator Centre
RDC	Research & Development Center
RSC	Rescue Sub-centre

S

SARP	Selective Assessment Routing Problem
SMC	SAR mission coordinator
SOG	Speed over ground
SOLAS	Convention for Safety of Life at Sea
SRR	Search and Rescue Region
SRRLX	Search and Rescue Region – Área de Lisboa
SRRST	Search and Rescue Region – Área de Santa Maria
SRU	Search and Rescue Unit
STPRBH	Steiner tree problem with revenues, budget and hop-constraints

T

TC	Tidal current
TDTSP	Time-dependent travelling salesman problem
TOP	Team orienteering problem
TSP	Traveling salesman problem

V

VHF	Very High Frequency
VMS	Vessel Monitoring System
VTS	Vessel Tracking System

U

UAV	Unmanned aerial vehicle
UN	United Nations
UNCLOS	United Nations Convention on the Law of the Sea
USCG	United States Coast Guard
UTC	Coordinated Universal Time

W

WC	Wind current
WMO	World Meteorological Organization

Table of Notation

Symbol	Page	Description
$G = (V, A)$	p. 5	Directed graph where V is the set of nodes and $A = \{(v_i, v_j): v_i, v_j \in V, i \neq j\}$ is the set of arcs
c_{ij}	p. 5	Nonnegative profit associated with each arc (v_i, v_j)
Q_k	p. 5	Capacity of vehicle k
A_k	p. 5	Subset of feasible arcs used by vehicle k
L_k	p. 5	Limit in the distance that vehicle k can travel
$S_D(k)$	p. 5	Set of possible starting nodes (starting depot) for vehicle k
$E_D(k)$	p. 6	Set of possible ending nodes (ending depot) for vehicle k
S_i	p. 6	Set of nodes associated to costumer i (each costumer is represented by a cluster of nodes)
d_i	p. 6	Demand of costumer i (all nodes in the cluster S_i have the same demand d_i)
M	p. 62	Partial solution (also called "master solution") within the pilot method
Tmax	p. 63	Maximum time available used as a stop criteria within the pilot method
$\mathbb{H}$	p. 62	Pilot heuristic or subheuristic used within the pilot method to obtain a fully grown solution from a partial solution
$cmin$	p. 63	Best known cost of fully grown solutions within the pilot method
Γ	p. 66	Set of sequences of elements that can be included into the master solution within the pilot method
ξ	p. 66	Sequence of elements that can be included into the master solution within the pilot method
F	p. 77	Set of facilities for helicopters (airbases)
N	p. 77	Set of initial location of nearby ships
M	p. 77	Set of meeting locations used to transfer survivors and are use as the finish depot for nearby ships
R	p. 77	Set of replenishment ships

t_0	p. 78	Initial instant of the incident which is also the time the MRCC receives the alert
τ_{t_0}	p. 78	Geographic location of the incident specified in latitude and longitude degrees at time t_0 (it is also refered as datum)
$LW_{\tau_{t_i}}$	p. 78	Local wind on datum τ_{t_i}. It comprises the wind direction and speed on datum τ_{t_i}
LW_{dir}	p. 78	Direction from where the wind blows at datum τ_{t_i}
LW_{spd}	p. 78	Speed in knotsof local wind at datum τ_{t_i}
$f\left(LW_{\tau_{t_i}}\right)$	p. 78	Object's leeway caused by the wind blowing on the exposed surface of the object
$WC_{\tau_{t_i}}$	p. 78	Wind current on datum τ_{t_i}
T_M	p. 78	Mission period, or mission time window, which can also be stated as $T_M = t_n - t_0$
U	p. 79	Random variable with uniform distribution
γ	p. 79	Random variable that represents the variations on the leeway speed
α	p. 79, p. 80	Time step parameter that scales the total drift force applied to an object located on datum τ_{t_i}
$f(i,t)$	p. 81	Node index of object i at the time index t
$g(i)$	p. 81	Cluster index of node i (clusters are associated to the objects in the MMRO problem)
$t^{\circ}(i)$	p. 82	Time instant (date format) associated to the departure time from datum i
$tr(i,j)$	p. 82	Travel time between datum i and datum j and it is obtained dividing the distance between i and j by the cruise speed of the helicopter or nearby ship
res_time	p. 82	Rescue time required by the vehicle to recover the survivor
$A^h_{F,R}$	p. 82	Set of arcs associated to helicopters from depots to nodes representating the location of a replenishment ship
$A^h_{F,S}$	p. 83	Set of arcs associated to helicopters from depots to nodes representing a datum of a survivor or raft
$A^h_{R,F}$	p. 83	Set of arcs associated to helicopters from the location of replenishment ship to a node representing a depot
$A^h_{R,S}$	p. 83	Set of arcs associated to helicopters from the location of replenishment ship to a node representing a datum of a survivor or raft

$A_{S,F}^{h}$	p. 83	Set of arcs associated to helicopters from the datum of a survivor or raft to a depot
$A_{S,R}^{h}$	p. 83	Set of arcs associated to helicopters from the datum of a survivor or raft to the location of replenishment ship
$A_{S,S}^{h}$	p. 83	Set of arcs associated to helicopters between survivor or raft datum
$A_{N,S}^{ns}$	p. 83	Arcs from the nearby ship initial location to the datum of a survivor or raft
$A_{S,M}^{ns}$	p. 83	Arcs associated to nearby ships from the datum of a survivor or raft to a meeting location
$A_{S,S}^{ns}$	p. 83	Arcs associated to nearby ships between survivor or raft datum
D^{h}	p. 84	Distance matrix associated to helicopters between time-index nodes
D^{ns}	p. 84	Distance matrix associated to nearby ships between time-index nodes
ref_time	p. 86	Time required to refuel an helicopter from a replenishment ships
vsk	p. 88	Line vector with the indexes of the objects (cluster indexes) associated to a solution or partial solution for the MMRO problem
vss	p. 88	Line vector with the indexes of the nodes (of the layered graph) associated to a solution or partial solution for the MMRO problem
x_{ij}^{kpq}	p. 89	Decision variable that indicate whether helicopter k travels from node (i,p) to node (j,q), $x_{ij}^{kpq} = 1$ or not, $x_{ij}^{kpq} = 0$ (the indices i and j refer to the problem objects and the indices p and q refer to time stamps)
y_{ij}^{lpq}	p. 89	Decision variable that indicate whether nearby ship l travels from node (i,p) to node (j,q), $y_{ij}^{lpq} = 1$ or not, $y_{ij}^{lpq} = 0$ (the indices i and j refer to the problem objects and the indices p and q refer to time stamps)
d_{ij}^{kpq}	p. 90	Distance between node (i,p) to node (j,q) (the indices i and j refer to the problem objects and the indices p and q refer to time stamps)
r_{ij}^{pq}	p. 90	Distance in nautical miles gained by refuelling at node (i,p)
Aut_{k}	p. 90	Autonomy of helicopters in nautical miles

u_i	p. 90	Weight associated to object i (it can be the number of persons or their respective weight)
Cap_k	p. 90	Capacity of vehicle k (it can be the total number of persons it carries or the maximum weight)
$\varphi(k,s)$	p. 95	Merit function associated to a vehicle/survivor assignment that can be based on distance, expected time arrival (ETA) or profit between a vehicle k and a survivor or object s
$rc(k)$	p. 95	State variable used within a construcitive heuristic and pilot method that is associated to a vehicle k and indicates if that vehicle can still rescue a survivor or an object

Chapter 1

Introduction

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1 Introduction

1.1 Problem description

The International Maritime Organization (IMO) defines a Mass Rescue Operation (MRO) as “a Search and Rescue (SAR) operation that involves the need for immediate assistance to large numbers of persons in distress such that capabilities normally available to SAR authorities are inadequate” [1, Sec. 6.6]. Incidents that requires MROs occur less frequently than typical search and rescue operations but have a much higher potential for severe consequences. Recently, incidents like the Costa Concordia accident, which occurred after departing Civitavecchia in 2012, and the refugee migrations in the Mediterranean are examples of scenarios that may require MROs. These incidents may involve hundreds or thousands of persons in distress in remote and hostile environments. A large passenger ship collision, for example, could call for the rescue of thousands of passengers and crew in poor weather and sea conditions, with many of the survivors having little ability to help themselves.

From the vast possible number of incidents that may require a MRO, this work focus on those that occur in the maritime environment, particularly on ocean areas. Incidents that may require a Maritime Mass Rescue Operation (MMRO) are numerous and with different characteristics. One feature in common is the large number of persons to be rescued. The weather conditions play a key role in the MRO since these can influence the survivability of persons that may be in the water or the time required by a helicopter or ship to recover them. In the case where an incident occurs in ocean waters, persons in water (PIW) or survivors inside a liferaft will be subject to maritime drift. If the arrival of SAR assets to the incident location takes too long, the action of these forces will disperse the survivors and complicate their recovery. After receiving an alert on a possible emergency situation, the Maritime Rescue Coordinator Centre (MRCC) will respond to the incident through a sequence of activities which are grouped in five different stages (SAR stages): Awareness, Initial Action, Planning, Operations and Conclusion (see Chapter 3.2 “SAR Stages” in [2]). When there is confirmation of an emergency situation search and rescue units (SRU) are dispatched to the scene in order to conduct searches, rescue survivors and provide the necessary emergency care. This is done in the operations stage. When all survivors are rescued or the situation assessment does not justify further search operations, the SRU return to a location where they are debriefed, refueled and prepared for other missions, which corresponds to the conclusions stage.

The problem addressed in this paper deals primarily with the rescue activities on the operations stage. The scenario involves an incident in ocean waters (typically a large passenger ship or large passenger aircraft) where there are survivors in the water or in liferafts and the environmental conditions will cause them to drift and disperse. The incident may be caused by a ship collision, terrorist attack or a sea ditch if we are considering an accident with an aircraft. The first hours after the alert is received in the MRCC are critical for the rescue operations success. For this reason we are interested in the first six to twelve hours prior to the incident alert time. The rescue operations will be performed by rotary wing crafts (helicopters) and nearby ships, which can operate simultaneously. We assume that the location of the survivors or rafts are known and also their drift through the 6 to 12 hour time period, which defines the mission time window. This means the SAR operation has only the "rescue" component. The "search" component is not part of the problem and it is assumed it is solved. Information regarding the survivability time for the survivors is a function of time and is used in the objective function of our problem. Survivability time depends on several factors. Survivors may be injured prior the incident, which can decrease greatly their health condition. If a person is in the water, the survivability time depends greatly on the water temperature (see figure N-14, annex N in [1]) and also the time it remains there, for there is a risk of hypothermia. Fatigue is another risk to take into consideration for it may lead to exhaustion and subsequently death by drowning. The effectiveness of a SAR operation can be measured by the number of lives saved regarding the total amount of lives available to be saved or by the value of preventing loss of property (see [2, pp. 5–13]). In our problem, only the prevention of loss of life is taken for measuring the rescue operation efficacy. One interesting feature of the mass rescue problem comprises multiple and simultaneously incidents that require MRO. Such cases are very rare but, even so, such scenarios have been recently a source of major concern for governments who have a high risk of organized terrorist attacks.

The maritime mass rescue operation (MMRO) problem will be viewed as a vehicle routing problem with a profit objective where vehicles, which can be either ships or helicopters, will recover the survivors at a specific location in time depending on their drift. The profit objective function is considered because it may not be possible to rescue all the survivors within the mission time window. There are distance constraints for helicopters depending on their operational range. Nearby ships that are dispatched to the incident location will terminate their route in a specific location designated by the MRCC in order to transfer survivors to SRUs capable of receiving them. This location acts as depot for nearby ships. These locations have to be considered because the ship's master is not obliged to divert course to the nearest and suitable port in order to disembark survivors.

Helicopters start their routing on a predetermined location, usually an airbase, and may terminate their routing on a set of possible locations. We also consider the possibility of special vessels that may refuel helicopters during the operation. Some helicopters have the capability of helicopter in-flight refuelling (HIFR) that extends their range and may increase the number of lives rescued. The location of such ships is predetermined as well as their trajectory through the mission time window.

1.1.1 Maritime Mass Rescue Operation problem in terms of graphs

In this subsection a graph-theoretic description of the MMRO problem is made. Our problem can be viewed as a special case of the Generalized Vehicle Routing Problem (GVRP) (see [3], [4]) with some borrowed features from the Prize Collecting Traveling Salesman Problem (PCTSP) (see [5], [6], [7]) and from the Multiple-depot Traveling Salesman Problem (MDTSP) and related variants (see [8], [9], [10]). The GVRP component of the MMRO problem is due to the sequence of positions in time of each object (due to the maritime drift). Each object (a single person in the water or a liferaft with fifty persons onboard) is represented by a cluster of nodes, where each node represents the location in time of that object. Therefore, we are only interested in visiting only one node in each cluster. The multiple-depot component of the MMRO problem has to do with the starting position of nearby vessels and airbases location. This implies that each vehicle (aircraft or vessel) may start their tour from different locations and may end in another different location from where they have started the tour. Hence, the need to consider multiple depots for starting the vehicle tour and also for ending it. In the prize collecting TSP it is not required that all nodes be visited by a vehicle. In a MRO where a time window is considered (in a sense where we are only interested in assessing what the SAR system can achieve in the first six to twelve hours following the first alert) it is expected that it may not be possible to retrive all the objects.

Let $G = (V, A)$ be a directed graph where V is the set of nodes and $A = \{(v_i, v_j): v_i, v_j \in V, i \neq j\}$ is the set of arcs. A nonnegative profit c_{ij} (in the MMRO problem represents the value for visiting the node v_i) is associated with each arc (v_i, v_j) and also a distance l_{ij} and an elapsed time value t_{ij} (represents the time elapsed from the clock start instant and the time associated with servicing node v_i and departing through the arc (v_i, v_j)). A fleet of K heterogeneous vehicles are available with capacity Q_k. Each vehicle can only use a subset $A_k \subseteq A$ of arcs (A_k can be represented by an adjacency matrix) and may have a limit in the distance L_k they can travel and an available work duration limit R_k (if the vehicle is a helicopter it is natural to consider a maximum flight duration). Each vehicle k has a fixed starting node $S_D(k)$ (starting depot) and a fixed set of possible ending nodes $E_D(k)$. The sets $E_D(k)$, $k = 1, \dots, K$, are not necessarily mutually exclusive. Let $S_D = S_D(1) \cup \dots \cup S_D(K)$ and $E_D = E_D(1) \cup \dots \cup E_D(K)$ be the sets of all starting depots and ending depots, respectively and $D = S_D \cup E_D$ be the set of all depots (starting and ending depots). Node set V is partitioned into $n+1$ nonempty subsets (or clusters) $D, S_1, S_2, \dots$, S_n. Each node in the set S_i, $i = 1, \dots n$, has the same demand d_i and $d_i \leq \max_k\{Q_k\}, i = 1, \dots, n$. Let $I_S = \{1, \dots, n\}$ be the costumers cluster index set and $w(i)$ be the cluster index of node $i \in V \backslash D$. The MMRO problem is to find a set of K paths, one for each vehicle, starting at their respective depot and ending at one of the possible depot alternatives, which maximizes the overall profit collected and satisfies distance and elapsed time constraints.

1.1.2 Example of MMRO problem and solution in graph

We illustrate the MMRO problem with the following example. The notation used is the one in the book of Wilson [11]. The MMRO problem has the following data:

- Set $V = \{v_0, v_1, v_2, \dots, v_{30}\}$ has 31 nodes and is partitioned into 8 clusters, which $n = 7$ clusters are costumers sets denoted by $S_i, i = 1, \dots 7$ and the depot set $D = \{v_0, v_1, v_2\}$. Let $I_S = \{1, \dots, 7\}$ be the costumers cluster index set.
- Sets $S_i, i = 1, \dots 7$ have the same number of nodes:
  - $S_1 = \{v_3, v_4, v_5, v_6\}$;
  - $S_2 = \{v_7, v_8, v_9, v_{10}\}$;
  - $S_3 = \{v_{11}, v_{12}, v_{13}, v_{14}\}$;
  - $S_4 = \{v_{15}, v_{16}, v_{17}, v_{18}\}$;
  - $S_5 = \{v_{19}, v_{20}, v_{21}, v_{22}\}$;
  - $S_6 = \{v_{23}, v_{24}, v_{25}, v_{26}\}$;
  - $S_7 = \{v_{27}, v_{28}, v_{30}, v_{31}\}$.
- All nodes in the cluster $S_i, i = 1, \dots 7$ has the same demand with: $d_1 = 50$, $d_2 = 2$, $d_3 = 5$, $d_4 = 1$, $d_5 = 20$, $d_6 = 6$, $d_7 = 20$.
- Two available vehicles with capacities $Q_1 = 15$ and $Q_2 = 75$, respectivelly. Let $L_k = \infty$ and $R_k = \infty$ for both vehicles (no tour length constraint nor tour duration limit are considered).
- Vehicle 1 start and end its tour on node v_0. Thus $S_D(1) = \{v_0\}$ and $E_D(1) = \{v_0\}$.

- Vehicle 2 start its tour at node v_1 and must end at vertive v_2 ($S_D(2) = \{v_1\}$ and $E_D(2) = \{v_2\}$).
- Consequently, $S_D = \{v_0, v_1\}$, $E_D = \{v_0, v_2\}$ and $D = \{v_0, v_1, v_2\}$.
- Set of arcs A does not contain arcs (v_i, v_j) if v_i and v_j belong to the same cluster.
- The matrix cost C can be any matrix, with adequate dimensions, with nonnegative values. The coefficient c_{ij} represents the human life value of the object when it is located at the node i. The human life value can be defined as a time dependent function.

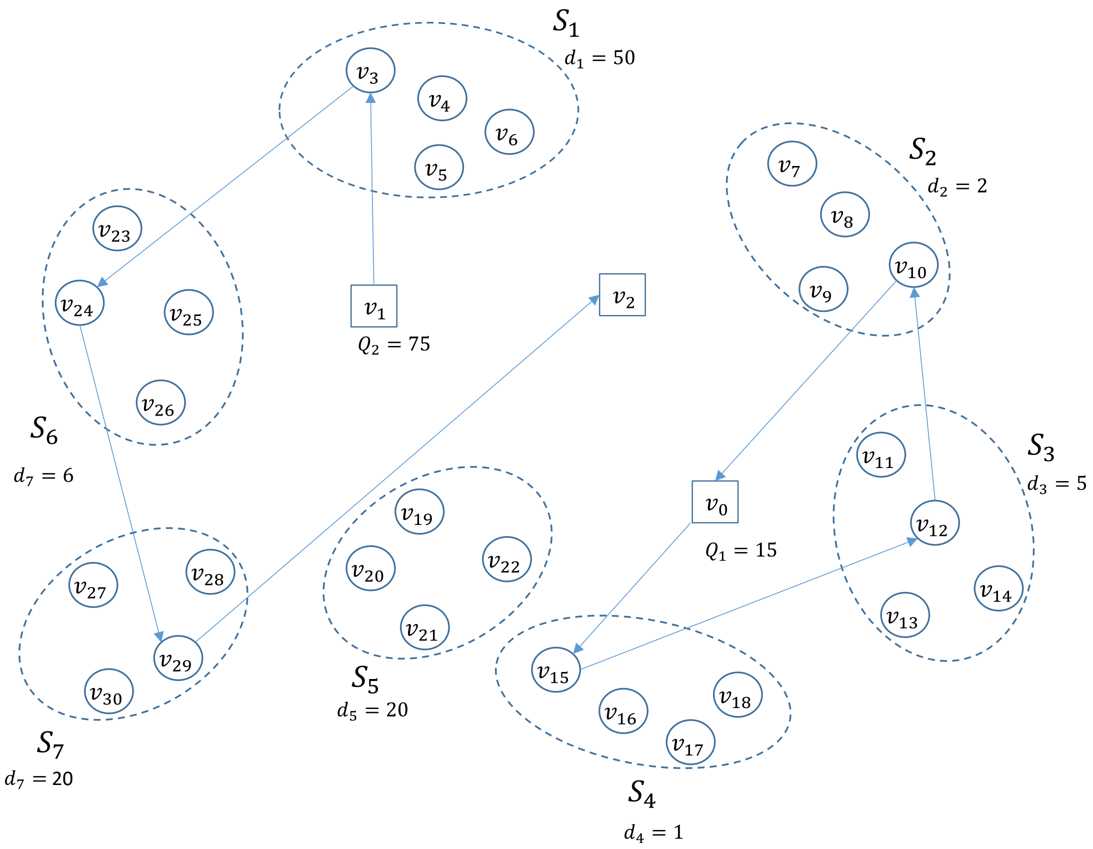

Figure 1. MMRO solution in the corresponding graph

In the solution illustrated in Figure 1, it is possible to observe that a vehicle's path can be a cycle. This can represent the tour made by a helicopter where it starts its tour from an airbase and returns there later. In the case of vessels, it is expected that they may desimbark survivors in a specific location, different from where they started their route. The example also shows that it is not mandatory to visit all clusters (cluster S_5 is not visited).

The example does not specify the conditions for allowing feasible arcs between nodes from different clusters. To model the object's track through time (sequence of positions of a SAR object due to the maritime drift) and to avoid having routes where vehicles "travel back in time", nodes will have a time stamp associated. Feasible arcs between clusters must satisfy the rule where the time stamp of the origin node is earlier than the time stamp of the destination node. This will be detailed in subsections 3.1.2 and 3.1.3.

1.2 Goals of the dissertation

This dissertation aims to answer the following question: "how to estimate the efficacy of a SAR system response to a mass rescue incident located in ocean waters using vehicle routing models?". If the "vehicle routing models" part were not specified in the question, then a possible answer could be implicitly found in the IAMSAR manual [1, Sec. 5.6] that states how the efficacy of a SAR system can be measured. Since the efficacy indicator is measured by the ratio between two quantities associated with the number of lives involved in a certain mass rescue incident, the problem of estimating the SAR system efficacy is a statistical problem, that would be solved after the respective MRO is concluded. The challenge in answering this question stands with the complexity of the incident and its effects (number of persons alive after the accident, expected time of arrival of search and rescue units to the scene, SAR units recovery capacity, weather conditions and survivors drift, survival times, etc) and, most of all, the location of the incident itself. The distance of the incident location to SAR facilities is believed to have a crucial effect on the final rescue outcome. In this sense, using mathematical models to recreate possible incidents that would require a MRO at different locations and with specific effects is paramount for a Coastal State to assess their own maritime SAR capability. The first goal of this dissertation is to develop a tool to create instances of incidents that require an MRO and also to evaluate different solutions for those problems.

A vehicle flow based formulation is proposed to model a mass rescue operation in ocean waters. The fundamental questions that arise to such endeavor are related with the solutions characteristics and how these may, in fact, reproduce a real rescue operation and, most importantly, how can we recreate an incident using relevant data that meet the requirements for a mass rescue operation. For example, does the model cope with the dynamics of moving objects when they are subject to maritime drift? How the weather conditions are taken into consideration in the model (how they affect drifting objects and vehicle recovery times)? How to guarantee that the times required by vehicles for rescuing survivors or retrieving objects (rafts, lifeboats, etc.) from the water are realistic and comply with the SAR experts judgement? These are modelling questions that must be analyzed and given a proper answer.

A second goal of this dissertation aims to study the procedures to retrieve drifting objects from the water and how these are influenced by available information on the scene. The procedure is defined by the sequence of choices made by SRUs in rescuing several dispersed survivors, which is associated to a priority rule (for example, a living person has has more priority than a deceased one when choosing who is to be retrieved from the water by a rescue boat). Particularly, we are interested in perceiving how the availability of survival time's data can influence the overall efficacy of the rescue operation, if this information is available to be implemented in a rescue procedure at the tactical level (or on-scene level). Having multiple vehicles, conventional procedures for retrieving drifting objects from the water are based on the vehicle's speed to reach the object's location (assuming it can retrieve it). In this sense, the vehicles expected time arrival (ETA) to a specific object's location stands as the "conventional" criteria (or standard priority) for obtaining a vehicle route and the respective sequence of retrieved objects. What if survival times were "available" to vehicles? Would a similar procedure based on the survival times provide better rescue solutions? These questions require the assumption that there is available technology that would provide the SAR system with the knowledge of the persons survival times and location with great accuracy. To answer these questions several variants of heuristics approaches are investigated that incorporate the priorities used by SRU vehicles during the recovery operations. To assess the quality of the heuristics that make use of standard priorities or available survival times, a more sophisticated heuristic approach based on a look ahead method is used for larger instances that cannot be solved optimally.

The final goal of this dissertation is to estimate the impact of SAR units availability on the SAR system response efficacy. Other variables, such has the location of the incident and the weather conditions, have critical impact on the SAR system response efficacy. Using the MMRO problem as an instrument to create instances with different effects, located at different distances from the SAR system facilities and with different SRU availability, can provide interesting data to assess the SAR system capability to deal with incidents that require an MRO and also provide arguments to support strategic decision concerning the aquisition of SAR equipment.

1.2.1 Summary of goals

The goals of this dissertation are summarized as follows:

- Develop a prototype to create instances of a maritime mass rescue problem (MMRO) problem.
- Study a vehicle flow based formulation to solve the MMRO problem. One important aspect in using this approach stands with the size of the instances. The number of SAR objets and available vehicles limits the size of the MMRO instances that can be created and also solved.
- Study heuristic methods that replicate the priorities given to SRU for retrieving SAR objects from the water depending on available information, particularly survival times.
- Estimate the impact of the availability of SAR facilities on efficacy of the SAR system response.

1.3 Scope of the dissertation

Althought a MRO is considered a special SAR operation, the MMRO problem does not consider the search element. It is assumed that the location of the SAR objects are known at each moment in time. Search problems in the maritime environment have been studied since the Second World War and these problems are concerned with the allocation of search effort to maximize the probability of detection of a certain target.

The term "rescue" is defined in IAMSAR Manual [1, p. x] as "An operation to retrieve persons in distress, provide for their initial medical or other needs and deliver them to a place of safety". This means a rescue operation can be viewed has having three distinct activities. The first one consists in retrieving persons in distress. The second one consists in providing for their initial medical needs and, the last one, involves delivering the retrieved persons to a place of safety. This dissertation only deals with the first activity: retrieving persons in distress. The other activities are not covered in this work and can also be viewed as optimization problems in the context of Disaster Operations Management (DOM). The problem of retrieving persons in distress located in ocean waters and subject to the maritime drift is refered as the maritime mass rescue operation (MMRO) problem. This problem consists in finding the optimal routes for a fleet of heterogeneous vehicles that minimizes a time dependent objective function. A solution for the MMRO problem provides information regarding persons retrieved alived and those who were retrieved and were allready deceased. In this problem it is assumed that the survival times are known and a person is considered to be rescued alived if it is retrieved before the moment it passes away. With the survival time information for each SAR object, it is possible to estimate the efficacy of the MMRO solution. The survival times are deterministic parameters in the MMRO problem and specified by the user when creating an instance.

Heuristics will be used for two distinct purposes. The first purpose is to replicate the priorities used by vehicles to retrive objects from the water. Two types of “priorities” are investigated. The first type, refered to as “standard procedure”, the SRU retrieves the nearest person in distress. When several SRU are available, the priority states that a person is recovered by the SRU that can reach the person’s location in less time. The second type of priority considers the availability of information regarding survival times and these are used by the SRUs when choosing the object to retrieve from the water. The second purpose in using heuristics is to solve large instances that cannot be solved within a limited amount of time and where a higher quality solution is required. For this later case we investigate the pilot method, proposed by Duin and Voss [12,13], to solve the MMRO problem.

A prototype was developed using MATLAB language to create MMRO instances and test different heuristics to obtain solutions and assess their quality. The prototype uses several geographic information system (GIS) functionalities that allow the user to place a SAR object or a vehicle in a specific location on a map and calculate distances between objects to create a MMRO instance.

As a test bed for the MMRO instances, these were created within the Portuguese Search and Rescue Region (SRR), which is one of the largest SRR among European Coastal States. Most of the Portuguese SRR covers a great part of the north Atlantic Ocean. Oceanic and atmospherical data for estimating the object’s maritime drift is made available from the webservice Saildocs [14] which allows users to download Grib files produced by the NOAA/NCEP[1] forecast models. Grib files are computer-generated forecast files from a NCEP/NOAA computer, which are sent without review, and are offered on an as-is basis. Both Saildocs and the computer model itself are automated systems that provide grib data on a daily basis with a six hour interval update on the forecasts. The grib files used to create the MMRO instances were provided by the Portuguese Navy, who collects data for their SRRs on a daily basis. These files provide weather forecasts on a variaty of physical variables (wind speed, wind direction, pressure reduced to mean sea level, etc) measured on a grid between the parallels 10º and 45º north and the meridians 45º west and 5º east. The drift algorithm to estimate the objects drift through time was implemented in MATLAB and uses wind forecasts from Grib files. The drift algorithm is valid only for ocean waters since the wind current parameters available on the IAMSAR manual only apply for these case.

The SAR facilities corresponding to airbases used for the MMRO instances are Montijo airbase, Lisbon airport, located in the Portuguese Continent, and Porto Santo airbase,

[1] National Centers for Environmental Prediction / National Oceanic and Atmospheric Administration,.

located in Madeira archipelago. Two types of experiments are conducted in this dissertation. One experiment aims to answer the question whether information regarding survival times would influence positively the rescue efficacy or not. The second experiment consists in considering an hypothetical mass rescue incident with a large passenger cruise ship based on real data near Madeira island. To further understand the SAR system that operates on these SRR, a brief description of the Portuguese Maritime SAR System is included in Chapter 2.

1.4 Structure of the dissertation

Chapter 2 addresses the concept of maritime mass rescue operations and the mathematical models and heuristics used to model the MMRO problem. A brief description of the Portuguese maritime SAR system is made and special attention is given to vessels geospatial data that are used to create the instances of the MMRO problem. A survey on look-ahead methods with particular emphasis to the pilot method for combinatorial optimization problems are also presented.

Chapter 3 provides a description of the parameters of the maritime mass rescue operation problem and its data structures and how they are calculated. A binary linear programming formulation for a vehicle flow fomulation is presented. This formulation is based on a (huge) layered graph, discretized by time where arcs between time-indexed nodes are feasible only if the ships or helicopters are allowed to travel within the given time ranges. Computational results for several constructive heuristics and a pilot method variants are presented.

Chapter 4 presents a scenario based simulation where several instances of a MMRO problem are created based on real data regarding the location of a cruise ship during its transit between Funchal (Madeira Island) and Malaga (Spain) in April 2016. The consequences of the incident were designed and grouped into scenario variants in order to assess the efficacy of the SAR response using nearby vessels and the SAR helicopter stationed at Porto Santo airbase.

Chapter 5 discusses the heuristic performance based on the computational experiments and the limitations of the MMRO problem as an instrument to assess a SAR system efficacy to a MRO.

Finally, Chapter 6 summarizes our conclusions and suggests future research opportunities.

To facilitate the understanding of the concepts used in this dissertation, images, photos and diagrams are used in several figures. Whenever the content in a figure is not exclusively made by the dissertation's author, the source of the content is refered below the figure's caption.

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Chapter 2

Literature Review

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2 Literature Review

This Chapter presents a survey of sources that covers the concept of maritime mass rescue operations and mathematical models and heuristics for vehicle routing problems that are relevant for the problem at hand. The Portuguese SAR system is also refered and special attention is given to vessels geospatial data that are used to create the instances of the MMRO problem. A survey on look-ahead methods for combinatorial optimization problems is also presented to provide context and facilitate the understanding of the implemented method for the MMRO problem.

2.1 Maritime mass rescue operations review

In this Section we provide a short introduction to the concept of maritime mass rescue operation and its most prominent components issues that are relevant for understanding the vehicle routing model that is presented in Chapter 3. The IAMSAR manual and the MRO project are the two major sources of information regarding MROs.

The term "mass rescue operation" had its first formal appearance in the circular 31 "Guidance for Mass Rescue Operations (MROs)" elaborated by the International Maritime Organization (IMO) Sub-Committee on Radio Communications and Search and Rescue (COMSAR) [15]. This document provides guidance to SAR authorities, Rescue Coordinator Centers (RCC), ship-owners, shipping companies and shipmasters, in preparing for, and coordinating aspects of major incidents involving rescue of large numbers of persons in distress from ships or downed aircraft. It also introduces a definition of Mass Rescue Operation that IMO adopted in its IAMSAR manual and it is used by SAR agencies worldwide to the present day. IMO defines a mass rescue operation as "*a rescue operation that involves the need for immediate assistance to a large numbers of persons in distress such that capabilities normally available to SAR authorities are inadequate*" (see paragraph 6.15.1 in [2, pp. 6–14] and paragraph 1 in the annex of [15]). MROs are considered to be relatively rare low-probability high-consequence events compared to normal SAR operations. However, on a world-wide basis and with a larger time-frame, major incidents leading to the need for MROs are a certainty. In many of these incidents a proper response was never possible to be executed in time due to the severity of the incident. There are several types of incidents that can require a MRO. The IAMSAR manual recognises "*flooding, earthquakes, terrorism, casualties in the offshore oil industry and accidents involving releases of hazardous materials*" as examples of incidents (see paragraph 6.15.3

in [2, pp. 6–14]) that, because of their scale and magnitude, may require a response from SAR authorities for which they may not be prepared. The U.S. Coast Guard addendum to the United States National SAR Supplement to the IAMSAR Manual also considers, as possible scenarios that could lead to a MRO, events such as tornados, hurricanes, weapons of mass destruction incidents, and passenger ship or large airliner disasters [16, pp. 3–53].

From all the possible incidents that may require an MRO we are interested in those that occur in the maritime environment. These type of incidents are very different from those that occur on land. Survivors may be subject to the maritime drift and to other factors that can hinder their survivability. In this context, a maritime mass rescue operation (MMRO) is a mass rescue operation where the incident is located in a maritime area. Several factors can affect the rescue operation and the survivability of persons in distress following a maritime large-scale incident:

- The temperature of the water is a crucial factor for the survival time of persons that couldn't shelter in a liferaft or lifeboat.
- Evacuation from a vessel that has to be abandoned can prove to be a difficult task to accomplish safely. Panic may arise in crowded environments and cause injuries among running persons trying to reach a lifeboat or simply leaving the ship by any means possible.
- The recovery process is often difficult. For example, the size of the recovering ship may hinder survivors that have to climb or be lifted considerable distances to get aboard.
- Differences in relative movement between the recovery ship and the craft or people alongside may also prevent survivors that are alongside to get onto ladders, etc.
- The physical capability of those to be recovered: they may be able to do little or nothing to help themselves.
- The maritime drift may delay the recovery of survivors, whether they are in the water or in liferafts or lifeboats.

Alongside with the above difficulties associated with a maritime large-scale incident, a MRO will require capabilities that are not readily available to SAR authorities. This requirement poses different challenges to SAR authorities because they will have to put extraordinary measures into effect in order to deal with these large scale incidents. These challenges focuses on planning MRO, coordination and communication between different entities involved during rescue operations, gathering additional resources (involvement of nearby shipping). But it is not just a question of physical resources. The rarity and variability of MROs prevents personnel and SAR staff to become expert in them. Also, due to the rarity of such events it is very difficult for authorities to justify maintaining sufficient resources do deal with it "routinely" (see paragraph 2.5 in [17, p. 3]). This is why it is important that authorities recognize the risk and acknowledge the need to prepare for such type of incidents. The International Maritime Rescue Federation (IMRF) MRO project group recommends that authorities allocate planning and training resources to deal with MROs.

The next subsection presents three cases of mass rescue operations. All of the incidents happened near SAR facilities and the rescue operations were set in motion very briefly after the first call for help. This will not always be the situation if the incident is located in ocean waters, like the Atlantic Ocean, which is the primary concern of the Portuguese Navy. All of these cases are likely incidents to occur in ocean waters. In [18] several examples of MRO are presented focused on passenger ships accidents and also passenger aircraft ditching.

2.1.1 Examples of maritime mass rescue operations

Sinking of the ro-ro ferry Estonia

One of the most tragic accidents with ro-ro ferries was the sinking of MS Estonia in the Baltic Sea. The MS Estonia sank between about 00:55 and 01:50 (UTC+2) on September 28, 1994, during its transit from Tallinn (Estonia) to Stockholm (Sweden). The ship was carrying 989 people, where 803 were passengers and 186 were crew staff. The MS Estonia incident is well documented in the final report made by the Joint Accident Investigation Commission [19] that gathered experts from Finland, Estonia and Sweden to investigate the accident. The incident started in the ship's bow door on the car deck. At about 01:15 hrs the ship's visor[2] separated from the bow and tilted over the stem. The ramp was pulled fully open, allowing large amounts of water to enter the car deck. The Maritime Rescue Co-ordination Centre (MRCC) in Turku received the Mayday call around 01:24 hrs through radio communications and immediately initiated rescue efforts. About one hour after the ESTONIA had sunk, four passenger ferries in the vicinity arrived on the scene of the accident. Rescue helicopters were summoned and the first one arrived at 0305 hrs. The first ship reached the accident scene 50 minutes after the *1st Mayday call*, i.e. 20 minutes after the vessel sank. Four passenger ferries and the first rescue helicopter were on the

[2] There are two main types of bow access in ro-ro ferries: the bow visor and bow door. The bow visor allows the ship bow to articulate up and down providing access to the cargo ramp and storage deck near the water line.

scene within one hour and ten minutes of the sinking. During the next three hours six more vessels and six more helicopters arrived. Only 138 persons were rescued alive, from which 137 survived. 34 were rescued by ships (16 by the ferry Isabella using her evacuation slides), 104 by helicopters.

Figure 2. The ro-ro passenger ferry ESTONIA (left) and its route with site of accident (right)
Source: Adapted from [19, Ch. 1]

The poor weather conditions prevented nearby ships from rescuing more survivors. The search for and retrieval of bodies and objects in the water continued until 2 October, after which searches continued in connection with the regular patrol flights of aircraft and helicopters.

The helicopters found 92 bodies within the first days. A total of 17 helicopters and 29 vessels were dispatched to the scene for search and rescue operations. The search operations continued after no more survivors were found (approximately at 10:00 hrs).

The major conclusions regarding the rescue operation stated that the alarming of helicopters was late (due to distress traffic was conducted separately from MRCC Turku, and that there was only one person on duty at MRCC Turku, at MRCC Helsinki and at Helsinki Radio, respectively) and these had a major role in rescuing survivors from the water and floating liferafts and lifeboats. In this particular case, only one rescue operator was deemed insufficient for retrieving survivors to the helicopter due to the exhausting rescue work. Between the first helicopter arrival and the rescue of all survivors there is a time window of approximately 6 hours, where the first hours are the most critical for rescuing persons alive. One of the difficulties in assessing the ESTONIA SAR operation efficacy rests in determining the number of persons that managed to survive the ship's sinking and could be eligible for rescuing. A total of 757 passenger and crew were known has missing persons (see Chapter 7.6 - Human outcome in [19]) and, from these, it is very difficult to tell who survived the sinking. The consequences of the accident reports to human casualties and the total loss of the ship.

The ESTONIA accident led to major changes in the SOLAS convention regarding ships' rescue capabilities and also its stability[3].

US Airways Flight 1549 - Aircraft ditching in the Hudson River

In January 15th, 2009, an airbus A320-214, with flight code US Airways Flight 1549, was struck by a flock of Canada Geese, just three minutes after takeoff from New York City's LaGuardia Airport and the cockpit crew managed to successful ditch the aircraft in the Hudson River. Immediately after the ditch, the crew began evacuating the passengers through the four overwing window exits and into an inflatable slide/raft deployed from the front right passenger door. The plane stayed afloat long enough for nearby ferries to assist the passengers and proceed to their rescue. All passenger and crew were evacuated from the plane in approximately 25 minutes. Fortunately, all 150 passengers and five crew survived, with only five serious injuries. The incident is well documented in the National Transportation Safety Board (NTSB) final Accident Report [20], which is the source of most the content presented here. The SAR facilities employed in the flight 1549 rescue consisted of seven New York Water Way (NY WW) ferries, one fire rescue boat and two USCG small boats. These vessels recovered all 155 passengers.

Figure 3. Flight 1549 aircraft surrounded by ferries and boats after evacuation

Source: ©CNN

[3] The most important changes concerned the stability of ro-ro passenger ships. A new regulation 8-1 of Chapter II-1 stated that existing ro-ro passenger ships will have to fully comply with the SOLAS 90 standard that was adopted for new ships in 1988. Ships that only meet 85% of the standard will have to comply fully by 1 October 1998 and those meeting 97.5% or above by 1 October 2005. A new regulation 8-2 was also adopted which requires that ro-ro passenger ships carrying 400 persons or more shall be designed to survive with two compartments flooded following damage. This regulation is also intended to phase out ships which carry 400 persons or more, built to a one-compartment standard of subdivision.

The rescue operation of passengers and crew did not had a search component, since it was not needed. Reports state that one passenger jumped into water, but was retrieved by one of the ferries. The final report concluded that the cause of the accident was due to the ingestion of large birds into each engine, which resulted in an almost total loss of thrust in both engines and the subsequent ditching on the Hudson River.

As a mass rescue operation, this incident reminds the risk of airplanes ditching in ocean areas. Airplanes with inflatable slide/raft in both aft doors may serve as temporary liferafts for persons who survive a hypothetical plane crash in ocean waters. Life jackets are also available under the passenger seats and these may prevent survivor's drowning if they are in the water. One of the differences between the Hudson River and the ocean is that in the latter case help may require more time to reach the scene. Another interesting fact was the ebb tide that moved the aircraft while in the water at a speed of 1.4 Kts. The tide drift did not halt the rescuing effort and all passenger were recovered alive. The rescue operation had a complete 100% efficacy, since all survivors (from the aircraft crash) were rescued alive and those with serious injuries recovered.

Costa Concordia grounding and partial sinking

Costa Concordia was an international passenger cruise ship operated by Costa Crociere Spa that grounded and partial sank near the island of Giglio on January 13th, 2012. The English translation of the Italian report [21] released by the Italian Ministry of Infrastructure (MIT) documents the facts and analysis made by the investigation body on the Costa Concordia accident.

The vessel was scheduled for an 8 day cruise in the Mediterranean Sea starting the voyage on the port of Savona (Italy) on January 7th. On January 13th at 20:18 local time (UTC+1), Costa Concordia left the port of Civitavecchia to face the last part of its cruise towards Savona with expected time of arrival at 07:12 hrs. While passing through the island of Giglio the ship hit the rock bottom of the sea, at 21:45 hrs, which made a breach 53 meters long on the port side. Later, it was verified that the breach had made opening in four watertight compartments (WC). Before the impact the ship's speed was over 16 Kts. The momentum of the ship when it hit the bottom caused the speed to halve regardless of the subsequent arrest of propulsion. After the first impact the ship drifted and was pushed back towards the island and finally grounded near shore at Punta Lazaretto (Giglio island). The damage made by the first impact caused the flooding to four contiguous compartments that caused a temporary power blackout when water flooded the engine room. According to SOLAS requirements, the ship could withstand flooding of two adjacent main compartments. The first alert was received by MRSC Livorno at 22:14 hrs by the Carabinieri of Prato (local police) stating that they had received the phone call from the mother of a passenger who reported the collapse of a portion of the ceiling of a room in the ship and passengers were ordered to wear life jackets.

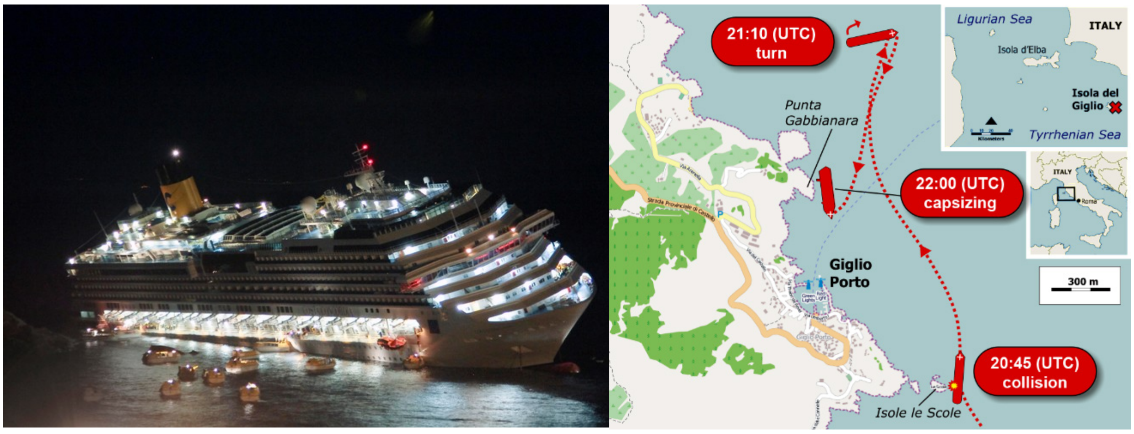

Figure 4. Costa Concordia during evacuation (left) and map with the route made near Giglio Island (right)
Source: adapted from Giuseppe Modesti/A.P. IMAGES and ©The Guardian

Subsequent communications between authorities and the ship were made to assess the circumstances of the accident. At 22:16 MRSC Livorno orders the immediate dispatch of a nearby patrol boat to the location of the accident which arrived at 22:39 hrs. The order to abandon ship was given at 22:54 hours while the ship was still drifting to shore. The ship had 4229 persons on board, where 3206 were passengers and 1023 were crew members. Approximately 2930 persons abandoned the ship using the ship's survival craft (boats and liferafts) and reached the coast by their own means. Nearly 1270 were rescued by the rescue units under the coordination of MRSC of Livorno, where 16 of these were rescued by helicopters. A total of 32 naval assets and 6 helicopters were dispatched to the scene [22]. From the 32 naval assets, 14 were merchant vessels and 4 were tug boats [21, p. 16]. The accident caused 32 mortal victims, 2 of which are still missing. Considering these values, one can state that the rescue operation had 99.24% efficacy. The resulting discussion at IMO was limited, as the cruise industry was agreed to be generally safe. Costa Concordia pre-dated the latest SOLAS amendments and the accident was considered to be caused by bad seamanship[4]. However, it showed that accidents do happen.

One key feature that distinguishes a mass rescue operation from a "normal" rescue operation is the involvement of nearby shipping. All of the three accidents presented above had the involvement of nearby ships whether they were specialized rescue units or private

[4] The accident was considered to be caused by human error during an attempt to make a spectacular parade past the island in what is known locally as an ''inchino'' or reverent bow, with its upper decks ablaze with light.

vessels. Nearby ships add to the recovery capacity necessary to retrieve a large number of persons in distress to a safer place. Usually, one of the "gaps" of the SAR system, when faced with a maritime large scale incident, is the lack of recovery capacity that the system can deploy in a short amount of time following the incident. In "normal" rescue operations it is frequent to commit nearby vessels to assist a distress situation, but usually there is no lack of recovery capacity and the situation won't require a prolonged assistance from a nearby ship.

The next subsection describes the legal obligations concerning rendering assistance to persons in distress at sea by vessels who are relatively near of the incident's location.

2.1.2 Legal obligations placed upon shipping for rendering assistance to persons in distress

In the mass rescue examples shown all the incidents had nearby ships (also called opportunity ships) and boats involved in the rescue operations that did not belong to the SAR authorities. Shipping in its broadest term can be any vessel, whether it's a fishing vessel, a leisure craft, a tanker, a cargo ship or a passenger vessel. The role of these ships in rescue operations can be critical because, in most cases, they are the ones who can reach the incident in the shortest time possible. Although there is a deep rooted moral obligation of helping those in need, and it is a long tradition for those who sail the seas to help fellow sailors, the ship's master decision to change course to save someone's life is not as simple as one may think. The current legal obligations placed upon shipping were never intended for the purpose of rescuing large numbers of persons. But providing assistance to any person in distress at sea is a clear legal requirement under international maritime law. The legal provisions concerning rendering assistance to persons in distress at sea are:

- United Nations Convention on the Law of the Sea (UNCLOS), 1982, Article 98(1) & (2);
- IMO International Convention for Safety of Life at Sea (SOLAS), 1974 as amended, Chapter V, Regulations 7 and 33;
- IMO International Convention on Maritime Search and Rescue (SAR), 1979 as amended.

The United Nations Convention on the Law of the Sea [23], which resulted from the third United Nations Conference on the Law of the Sea (UNCLOS III)[5], is an international

[5] English version at http://www.un.org/Depts/los/convention_agreements/texts/unclos/unclos_e.pdf

agreement that defines the rights and responsibilities of nations with respect to their use of the world's oceans, establishing guidelines for economic activities, the environment, and the management of marine natural resources. This convention also sets the geographic limits of various maritime areas, namely the territorial waters, contiguous zone, archipelagic waters and economic exclusive zone. Article 98 paragraph 1, states that "Every State shall require the master of a ship flying its flag, in so far as he can do so without serious danger to the ship, the crew or the passengers to render assistance to any person found at sea in danger of being lost and to proceed with all possible speed to the rescue of persons in distress (…)" [23, p. 60]. Paragraph 2 of the same article states that "Every coastal State shall promote the establishment, operation and maintenance of an adequate and effective search and rescue service regarding safety on and over the sea and, where circumstances so require, by way of mutual regional arrangements cooperate with neighbouring States for this purpose.". The first paragraph states that a ship's master is obliged to render assistance to persons in distress, if such is demanded by the State SAR authorities, but only if such request does not seriously endanger the ship's crew or passengers.

The International Convention for the Safety of Life at Sea (SOLAS) is an international maritime treaty that ensures ships flagged by Signatory States to comply with minimum safety standards in construction, equipment and operation. The first version was adopted in 1914, in response to the Titanic disaster[6]. There were three more versions before the 1974 version, which included a tacit acceptance procedure (which provides that an amendment shall enter into force on a specified date unless, before that date, objections to the amendment are received from an agreed number of Signatory States). As a result the 1974 Convention has been updated and amended on numerous occasions [25, pp. 11–68]. The Convention in force today is referred to as **SOLAS, 1974, as amended** [26]. The last version of the treaty includes several articles setting out general obligations followed by an annexe divided into fourteen Chapters. Of these Chapters, Chapter V (Safety of Navigation) is the only one that applies to all vessels on the sea, including private yachts and small craft on local trips as well as to commercial vessels on international voyages.

[6] RMS Titanic was a passenger liner that sank in the North Atlantic Ocean in the early morning of April 15, 1912 after colliding with an iceberg during her maiden voyage from Southampton, U.K., to New York City, U.S. The sinking resulted in the loss of more than 1,500 passengers and crew. The disaster caused worldwide shock and outrage due to the huge loss of life. Public inquiries in Britain and the United States [24] led to major improvements in maritime safety. One of their most important legacies was the establishment in 1914 of the International Convention for the Safety of Life at Sea (SOLAS).

Chapter V presents thirty five regulations that obliges governments to ensure that all vessels are sufficiently and efficiently manned from a safety point of view. Regulation 7 (Search and rescue services) requires States to ensure that necessary arrangements are made for co-ordination of SAR operations within their area of responsibility. Regulation 33 (Distress messages: Obligations and procedures) adds an obligation for all vessels' masters to offer assistance to those in distress and the terms to their release from the obligation to render assistance.

It is important to note that requisitioned vessels to assist persons in distress are only required to "assist" them. This implies retrieving persons in distress and provide for their initial medical or other needs. It does not require that the vessel should deliver them to a place of safety different from the ship's destination. This means that ships, in particular commercial ships, are not obliged to change their port of destination to disembark survivors. The responsibility for rescuing persons in distress, in the sense of retrieving, providing medical assistance and deliver them to a place of safety, is an obligation of the SAR service responsible for coordinating rescue operations in the area where an incident has occurred. While in the UNCLOS the obligation to guarantee an effective SAR service is set on the States authorities, in the SOLAS convention the obligation to render assistance to persons in distress is set on shipping flying the Signatory State's flag.

The IMO International Convention on Maritime Search and Rescue (SAR), 1979 as amended [27], consists in an international treaty aimed at developing an international SAR plan, so that, no matter where an accident occurs, the rescue of persons in distress at sea will be coordinated by a SAR organization and, when necessary, by co-operation between neighboring SAR organizations. In this convention, States agree for the provision of adequate SAR services in their coastal waters and are encouraged to enter into SAR agreements with neighboring States involving the establishment of SAR regions, the pooling of facilities, establishment of common procedures, training and liaison visits. The technical requirements of the SAR Convention are contained in an Annex, which was divided into five Chapters. Regarding the requisition of nearby vessel to assist in rescue operations, the 2004 amendment adopted by the Maritime Safety Committee (MSC), specified in paragraph 3.1.9, Chapter III, that "(…) ships by embarking persons in distress at sea are released from their obligations with minimum further deviation from the ships intended voyage, provided that releasing the master of the ship from these obligations does not further endanger the safety of life at sea" [28]. This paragraph strains the role of nearby ships in rescue operations and their relevance to act to relieve persons in distress, but when it is clear that no more danger to life exists, they should be released from further obligations. These include disembarking survivors to a nearby port of convenience different from the ship's port of destination.

In a situation where it is not possible to transfer survivors to the closest safe location after the incident and there is no threat to life, survivors are disembarked in the ship's destination port. It may be the case that the destination port may be much further away from the closest safe location relative to the incident's scene. If it is achievable, SAR authorities may provide additional resources to facilitate the survivors transfer to a rescue unit or to a specific vessel requisitioned for such task, which may take them to the nearest place of safety in land. Such transfer can only be made if the right conditions are met (appropriate weather conditions for a safe transfer, adequate health state of survivors). This situation has impact in the modelling of the vehicle routing problem (presented in Chapter 3) since requisitioned vessels will be regarded as vehicles in which their **ending depot will be a specific location at sea** where we assume the transfer of retrieved survivors will occur (see subsection 3.1.2). Another relevant issue regarding nearby ships who are required to participate in rescue tasks is the fact that **no monetary cost will be payed for their effort**. This means that there will be no fixed cost associated with using a nearby ship as a vehicle in the MMRO model.

The next subsection discusses search and rescue operations doctrine based on the IAMSAR manual which is the main source of content regarding the implementation and organization of a SAR system and the co-ordination and execution of search and rescue operations. It is also important to present the recommendations for evaluating the SAR system effectiveness which depends on the SAR operations efficacy. Due to the nature of mass rescue operations being low probability/high consequence events, it is relevant to mention the current efforts that SAR related organizations are undertaking regarding MROs.

2.1.3 Mass rescue operation doctrine

Maritime mass rescue operations are a particular case of a search and rescue operations. The key source of material regarding search and rescue operations is contained in the International Aeronautical and Maritime Search and Rescue Manual (IAMSAR Manual). Questions such as "how to plan a SAR operation?" and also "how to implement an efficient SAR system?" are addressed in the IAMSAR manual.

The IAMSAR Manual is a joint publication by IMO and the International Civil Aviation Organization (ICAO[7]) and its primary purpose is to assist States in meeting their own search and rescue (SAR) needs, and the obligations they accepted under the Convention on International Civil Aviation [29], the International Convention on Maritime Search and Rescue and the International Convention for the Safety of Life at Sea (SOLAS). The IAMSAR manual is divided into three volumes:

- Volume I, Organization and Management [1], discusses the global SAR system concept, establishment and improvement of national and regional SAR systems and co-operation with neighboring States to provide effective and economical SAR services.
- Volume II, Mission Co-ordination [2], assists personnel who plan and co-ordinate SAR operations and exercises.
- Volume III, Mobile Facilities [30], is intended to be carried aboard rescue units, aircraft and vessels to help with performance of a search, rescue or on-scene co-ordinator function, and with aspects of SAR that pertain to their own emergencies.

The topic of "Mass rescue Operations" is introduced in the Volume I, Chapter 6 [1, pp. 6–8], where the definition of MRO is presented along with general guidelines[8] for SAR authorities to deal with such type of operations.

One of the most important concepts presented and detailed in the IAMSAR manual is the concept of "global SAR system". The global SAR system is comprised by each signatory State regional SAR system which cooperate together to achieve an improved SAR service independently from the location of an incident. Each SAR system has individual components that must work together to provide the overall service. A complete description of the SAR system components can be found in [1, pp. 2–1 to 2–12]. One of these components are the Search and Rescue Regions (SRR) which, normally, are controled by the SAR services of a specific coastal State. Development of a SAR system typically involves establishment of one or more SRRs, along with capabilities to receive alerts and to co-ordinate and provide SAR services within each SRR. Each SRR is associated with

[7] The International Civil Aviation Organization is a specialized agency of the UN responsible for coordinating and regulating international air travel. Before ICAO became a UN specialized agency in 1947, it functioned has an independent and autonomous agency following to the signature of the "Chicago Convention" (more commonly known name for the Convention on International Civil Aviation) in 1944. More information on ICAO can be found in the organization's website www.icao.int.

[8] The presented guidelines focus on the development of MRO plans and exercices, joint cooperation between SAR agencies and companies that operate aircraft and ships designed to carry large numbers of persons and. It also recommend to provide information to the media without delays.

an Rescue Co-ordination Centre (RCC) or, if necessary, one or more rescue sub-centres (RSC) to support an RCC within its SRR. Aeronautical SAR responsibility may be met by means of an aeronautical RCC (ARCC). Coastal States with the added responsibility for maritime SAR incidents can meet this with a maritime RCC (MRCC). A list of required and desired capabilities that should be provided by an RCC is listed in [1, pp. 2–5, table 2.2]. Each RCC is supervised by a SAR mission coordinator (SMC) who will be responsible for directing and supervising a SAR operation until a rescue operation has been successfully concluded or until it has become apparent that further efforts would be of no avail, or until responsibility is accepted by another RCC. When multiple SAR units are working together on the same mission, one person is assigned to co-ordinate activities of all participating units. This is done by the On-scene co-ordinator (OSC). The OSC is designated by the SMC and usually the person choosen for OSC has proven experience in rescue operations and is aboard a vessel with suitable communications capabilities. It is natural that the nominated OSC changes to another person if a more specialized unit (such as a military unit or SAR units) arrive to the scene. The functions of the OSC are described in [2, pp. 1–3].

The IAMSAR manual considers that the response to a SAR icident usually proceeds through a sequence of five stages [2, pp. 3–1 to 3–2]: “Awareness”, “Initial action”, “Planning”, “Operations” and “Conclusion”.

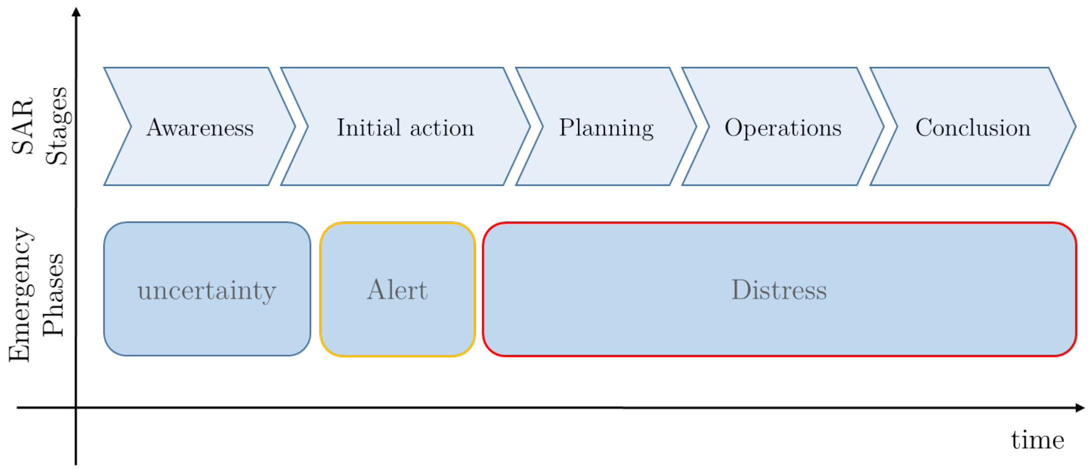

Figure 5. SAR stages and emergency phases during SAR incident

In each stage a series of activities are performed by the SAR system in response to a SAR incident from the time the system becomes aware of the incident until the response is concluded. IMO International Convention on Maritime Search and Rescue (SAR), 1979 as amended, established three emergency phases to classify incidents regarding the degree of emergency and help in determining the actions to be taken for each incident. These are the “uncertainty phase”, the “alert phase” and the “distress phase”. A thourough description of the emergency phases can be found in [2, pp. 3–2 to 3–3].

In the awareness stage the SAR organization has received information of a potential emergency situation, but it is not yet confirmed. This can be caused by the report of a missing person or difficulties in contacting a ship or aircraft but the information is not yet certified as conclusive for further action. Reports of overdue situations of ship’s or persons can prompt initial actions to confirm a potential emergency situation. In the initial action stage the RCC proceeds to evaluate and classify the information received regarding a possible emergency situation. RCCs usually have a checklist of steps to accomplish for each type of incident with which the RCC expects that it may become involved. It is at this stage that the SMC should declare the appropriate emergency phase and immediately inform all appropriate centres, personnel and facilities. The planning and operations stages can occur simultaneously. Planning activities such as calculating search areas or rescue plans (who is going to recover who) may be performed multiple times according to the development of the situation while search and rescue operations are being conducted. “Chapter 4 - Search planning and evaluating concepts” and “Chapter 5 - Search techniques and operations” of [2] provides a description of basic concepts of search theory and also search techniques and procedures to guarantee an efficient allocation of the search effort. The methods for optimal search effort allocation presented in the IAMSAR manual derive from the work developded by the Allied Antisubmarine Warfare Operations Research Group (ASWORG) during World War 2, in particular the work on optimal search planning for submarine detection by Koopman [31]. More recent developments in Search Theory can be found in the book of Stone [32] and Chudnovsky [33]. The conclusion stage corresponds to the return of the SAR units to their bases where they are debriefed, refuelled, replenished, and prepared for other missions, and to the return of other SAR facilities (requisitioned commercial vessels, fishing ship’s, etc) to their normal activities, and completion of all required documentation.

System effectiveness and efficiency

When establishing a SAR service, States should measure the effectiveness of the SAR system regarding the objective to minimize loss of both life and property. The IAMSAR manual proposes two measures that relate the SAR system effectiveness to the primary benefits to the population it serves. These measures are presented in paragraph 5.6.7 in [1, pp. 5–13]:

Programme effectiveness for preventing loss of life $= EFF(L) = \dfrac{LS}{LS + LLA}$ (2.1)

Programme effectiveness for preventing loss of property $= EFF(P) = \dfrac{PLP}{PLP + PL}$ (2.2)

where:

EFF	*Effectiveness*; L – *Lifes*; P –*Property*
LS	*Lives Saved*
LLA	*Lives Lost After Notification*
PLP	*Value of Property Loss Prevented*
PL	*Value of Property Lost*

Although the IAMSAR manual recomend these indicators to measure the effectiveness of the SAR system, they can be used to measure the effectiveness of a specific component such as the effectiveness of the maritime SAR service. In both measures, the denominator represents the total lives or property available to be saved. The numerator represents the lives or property actually saved. The resultant ratios measure the proportion of lives or property actually saved versus the total available to be saved. Lives lost before SAR system notification are not considered eligible to be accounted in the denominator of EFF. Therefore, they are excluded from the life-saving effectiveness measure. Lives lost after notification (LLA) reflect the potential number of additional lives that may have been saved. This number can be very difficult to estimate due to the lack of information regarding possible survivors after an incident and subsequent notification to the SAR system. In the example of the Estonia sinking, when the SAR system was alerted, all passengers and crew where alive. But in the Costa Concordia case, there is no information about the 33 deceased or missing persons during the time period between the first collision and the first notification.

The effectiveness measures presented should be sensible to modifications to the SAR system. Improved response times by the SRU and improvements in the awareness capability are expected to improve the SAR system effectiveness. Nonetheless, the measures are also affected by external factors to the SAR system. Initiatives like aviation or boating safety initiatives, or legislation to reduce drinking of alcoholic beverages, improved surveillance by life guard in beaches during sumer periods, should reduce the number of people and property in need of saving.

The U.S. Coast Guard addendum to the United States National SAR Supplement to the IAMSAR Manual [16, p. PPO-4 to PPO-6] proposes two measures similar to (2.1) and (2.2). While measures (2.1) and (2.2) focus exclusively on the response efforts of the SAR system, the proposed measures in [1] are intended to measure the effectiveness of the collective prevention and response efforts. When a live is in distress there are three possible outcomes for a rescue operation: the life is saved, the life is lost or the person remains missing after the conclusion of the SAR operation. The portion of "lives lost" can be divided in "lives lost before notification" and "lives lost after notification". Missing persons are not divided in "after" or "before" notification. The following equations encompasses the effectiveness of the total search and rescue system, response and prevention activities:

$$\text{Effectiveness of the total search and rescue system, response and prevention activities} = \frac{LS}{LS + (LLB + LLA + LUF)} \tag{2.3}$$

$$\text{Effectiveness of prevented loss of property} = \frac{PS}{PS + PL + PUF} \tag{2.4}$$

where:

LLB	*Lives Lost Before Notification*
LUF	*Lives Unaccounted For*
PUF	*Property Unaccounted For*

Cost-benefit ratios can be determined and used to measure the SAR system efficiency. However, it is difficult to determine the general value for a human life. Paragraph 5.6.13 in [1, pp. 5–13] proposes an approach that relates the effectiveness of saving lives to the total direct SAR costs for a given year, as shown below.

$$\text{Program effectiveness} = \frac{EFF(L) \times 100{,}000}{direct\ SAR\ program\ costs} \tag{2.5}$$

The scaling factor (100,000 in this example) is used to eliminate excessively small numbers.

The measures presented can be used to assess the effectiveness of a single rescue operation. The cumulative average during a civil year represents the effectiveness of the SAR service in that specific time period.

IMRF MRO project

The MRO project (www.imrfmro.org), lead by the International Maritime Rescue Federation (IMRF), provides MRO guidance directed to the SAR communities and promotes conferences to discuss relevant SAR related issues and SAR training courses envolving neighboring Coastal States personnel. The Gotenburg series of conferences, held in Gotenburg, Sweden, is one of the leading initiatives that allows SAR communities to share ideas, experience and prepare for large scale incidents. These biennial conferences have a workshop which includes a tabletop exercice that allows stakeholders to train and discuss the challenges that commonly arise in MROs. Reports describing the major results of the conferences are sent to the International Maritime Organization (IMO) (see [34–36]). One of the major contributions of the MRO project is the development of an online library (see [37]) of relevant information intended to raise awareness for the challenges posed by MRO. The guidance papers are grouped into five primary subject areas, all available online:

- **"Philosophy & Focus"** (see [17], [38–40]). This category focus on the challenges that large scale incidents present and the need to recognize the risk and the importance of planning and trainning resources to deal with such events.
- **"Planning"** (see [41–48]). Due to the complexity of MRO and its low probability and high consequence nature, planning is deemed as a way to identify capability gaps in the coastal state's SAR system and the means to filling them. It also sets the terms for wich exercices can be planned and executed.
- **"Resources"** (see [49–52]). Since the resources "normally available" are not adequate for coping with a MRO, these guidance papers focus on alternative ways to plan for additional SAR resources, whether these are vessels or shoreside facilities.
- **"Command, Control, Coordination, Communication"** (see [53–61]). Communication is one of the critical aspects in any rescue operation, specifically during its execution. These papers discuss relevant issues related with the need for increased coordination between all those involved in the rescue operation at different levels of action: "operational" level, "on-scene" level and "tactical" level.

- **“Training, exercises / drills, and learning from experience”** (see [62–66]). These papers focus on training MRO planning and how to exercice those plans. IMO guidelines for SAR training [67] are also referenced in these subject area.

One of the recommendations made by the MRO project emphasize **the need for organizations to assess their SAR capability** (see paragraph 4.2 in [40]) and have it mapped so that it may contribute to risk analysis and MRO planning. The recommendation does not specify “how” to do this, but the training guidelines from MSC Circular 1186 [67] focus on specific subject items[9] that should be addressed and assessed in order to perceive how the SAR system will respond to a large scale incident. The model presented in Chapter 3 considers some of those subject items, which are considered as parameters that describe a large scale incident, and presents a method to estimate the overall efficacy of the SAR system when faced with such an incident. Some of the “subject items” can be modelled to describe the scenario for a large scale incident that requires an MRO, such has the “survival times”, “location of the incident” and “number of survivors alive” subsequent to the incident. This is detailed in Section 3.1 in Chapter 3.

2.1.4 Portuguese Maritime SAR System

The International Civil and Aeronautical Organization (ICAO) and the International Maritime Organization (IMO) are the UN agencies responsible for coordinating and assisting the efforts of signatory States to implement and improve their aerial and maritime search and rescue services respectively. Their primary goal is to achieve a global SAR System, so that wherever a person is flying or navigating there is an available SAR service to render assistance if it is necessary. These services comprehend the capability to monitorize communications (particularly channels dedicated for emergencies communications[10]), coordinating search and rescue operations, providing advice and assistance, and also providing medical evacuation.

Portugal is one of the signatory States of the SOLAS convention. Through the decree-law n.º32/85 of 16 August 1985, Portugal ratified the SOLAS convention and commited itself to cooperate with other nations who have similar SAR responsabilities and also to

[9] The subject items considered in complex incident training are: “recognising the scale of the incident”, “survival time”, “SAR facility availability”, “working with strangers”, “mutual awareness”, “coordination overall”, “on-scene coordination”, “information, and lack of information”, “communications”, “language difficulties”, “planning and plans”, “priorities”, “recovery / retrieval of people in distress”, “counting those recovered”, “dealing with survivors”, “dealing with the injured”, “dealing with the dead”, “places of safety”, “news media interest”, “friends and families”, “logistics”, “politics: who's in charge?”, “fatigue”, “stress”, “training and exercising” and “leassons learned”.

[10] For example, the International Telecommunications Union (ITU) has established VHF channel 16 (156.8 MHz) as a distress, safety and calling channel, and it is monitored 24 hours a day by many coast guards around the world.

use the IAMSAR manual as guidance. In 1994, the decree-law n.º 15/94 of 22 of January creates the Maritime Search and Rescue National System[11] (SNBSM), refered in this thesis as the "Portuguese Maritime SAR System", which takes into consideration several statutory measures to ensure the establishment of an adequate structure, organization and functions of the SNBSM that guarantees the accomplishment of the objectives set by the IMO International Convention on Maritime Search and Rescue (SAR), 1979 as amended. The SNBSM involves a set of services and entities responsible of guaranteeing the safety of life at sea, as well as its procedures. The SNBSM is directed by the Ministry of Defence (MoD) who is supported by a consultive commision formed by representatives of different Ministries of the Portuguese Government (for more information about the Portuguese Government see [68]) and comprises military and civilian entities. The next figure shows the two complementary structures in which the SNBSM is organized: the principal structure and the auxiliary structure.

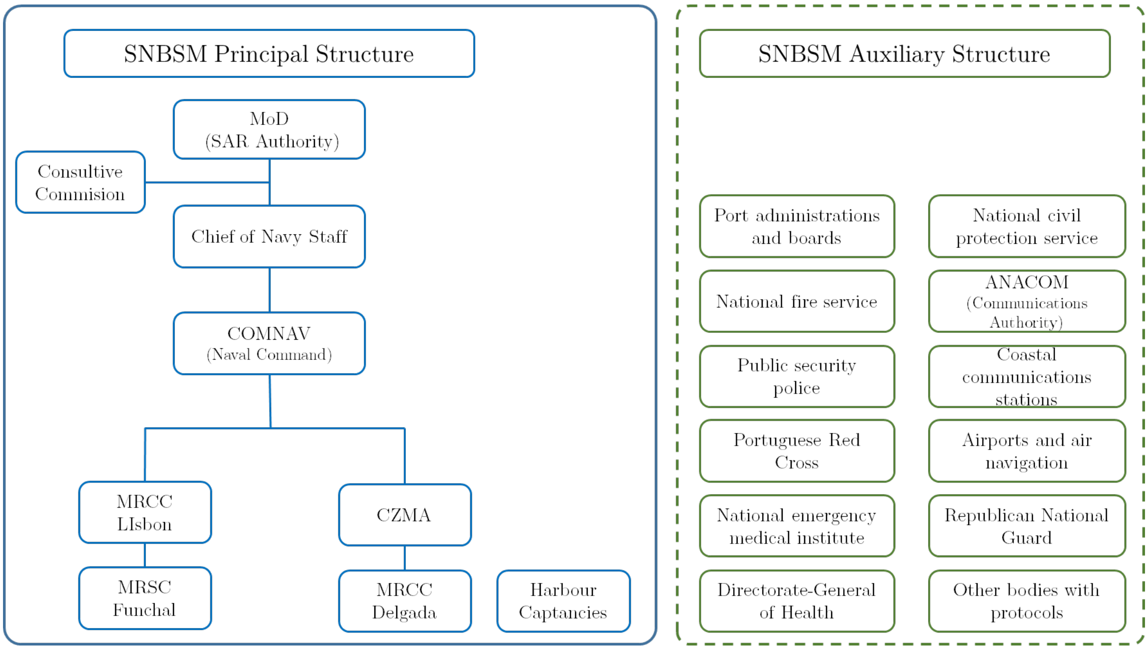

Figure 6. Organization chart of the Portuguese Maritime SAR System

Source: Adapted from [69]

The Maritime Search and Rescue Service (Portuguese Maritime SAR Service) is guaranteed by the Portuguese Navy[12] (PO Navy) [70, 71] who is responsible for conducting

[11] The translation in portuguese stands for "Sistema Nacional para a Busca e Salvamento Marítimo" (SNBSM). For simplicity we shall use the portuguese acronym throughout this dissertation.

[12] The navy of Portugal is commonly referred to as the "Portuguese Navy" both in Portugal and other countries and is one of the Armed Branches of the Portuguese Armed Forces, alongside with the Portuguese Army and the Portuguese Air Force. The Portuguese Navy is headed by the Chief of Navy Staff and includes the Navy Staff, the Personnel, the Material, the Finance and the Information Technologies superintendences, the Naval Command (naval component command, with

search and rescue operations concerning incidents involving vessels or persons at sea. The bodies that incorporate the Maritime SAR service are:

- Maritime Rescue Co-ordinator Centres (MRCC)
- Maritime Rescue Co-ordinator Subcentre (MRSC)
- Coastal surveillance units
- Search and rescue units (SRU)

Portugal, through the Portuguese Navy, is responsible for providing the maritime SAR service within two Search and Rescue Regions (SRR): the Search and Rescue Region of Lisbon (SRR Lisbon) and the Search and Rescue Region of Santa Maria in Azores (SRR Santa Maria). The figure bellow depicts the portuguese SRRs and the location of their respective MRCCs.

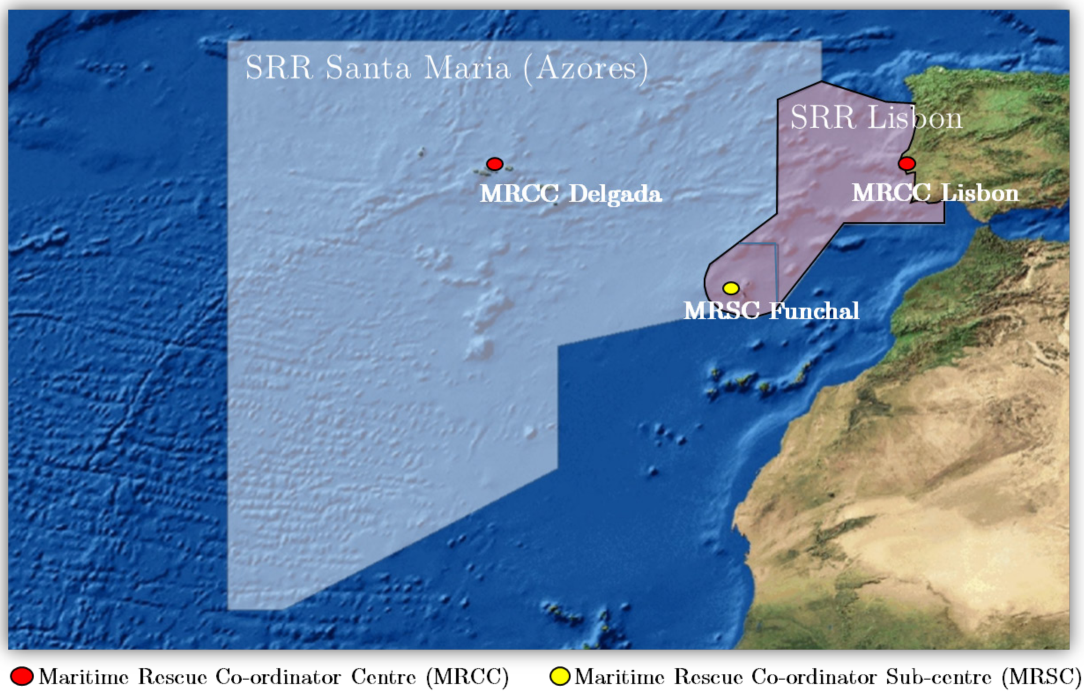

Figure 7. Portuguese Search and Rescue Regions (SRRs)

Source: adapted from ©Portuguese Navy

Both SRRs are situated in the Atlantic Ocean and together they cover a total of 5.8 millions square kilometers, which is the largest SRR among European Coastal States. Each SRR has a dedicated MRCC. The MRSC Funchal depends functionaly from the MRCC Lisbon and its area of responsibility is set south from the parallel 35º00'N and west from

five subordinate maritime zone commands), the Council of the Admiralty and the Inspection-General of the Navy. The decree--law n.º185/2014 of 29 December defines the structure and organization of the Portuguese Navy.

the meridian 015º00'W. MRSC Funchal guarantees the co-ordination of SAR operations within its area of responsibility led by the directives received from MRCC Lisbon. MRCC Lisbon and MRCC Santa Maria are functionally dependent of the Naval Command and the Azores Maritime Zone Command[13], respectively.

The coastal surveillance units refers to the coastal surveillance posts and the maritime traffic control centres. The naval search and rescue units of the SNBSM are provided by the Portuguese Navy are they comprise several types of vessels. Each SRR has a permanent naval SRU and one in reserve. Currently, the Portuguese Navy operates the Ocean Patrol Vessel (OPV) Viana do Castelo, the Baptista de Andrade class and Jacinto Cândido class corvettes and, more recently, the Tejo class offshore patrol vessels. All of these classes have an endurance greater than 15 days at sea and can operate under rough sea conditions[14].

Figure 8. Naval SRU types employed by the Portuguese Navy in the SRRs. From left to right: Viana do Castelo class ocean patrol vessel, Baptista de Andrade class corvettes and Tejo class offshore patrol vessel
Source: ©Portuguese Navy

For ocean areas within both SRRs, the mission coordination will be assumed by the Naval Command and its respective MRCC. Usually the MRCC operations officer will act as the SMC. In coastal areas, following the report of an incident within the area of responsibility of a certain harbour captancy, the respective harbour captain will assume the functions of SMC until the MRCC assume that function.

Portugal also ratified the Chicago Convention in 1947 through the decree-law n.º36 158 of 17 February. In 1995 this decree-law was replaced by the decree-law n.º253/95 of 30 of September which created the Aerial National Search and Rescue System (SNBSA). The organization is very similar to the SNBSM (it also presents a principal and an auxiliary structures), but the Portuguese Air Force is the Portuguese Armed Force branch responsible for providing the aerial SAR service within two Flight Information Regions

[13] The Naval Command (COMNAV) is the main operational command in the Portuguese Navy responsible for the conduct of naval operations with the purpose of ensuring that the Portuguese use the sea in the measure of their interests, supporting the exercise of command of the Chief of Navy Staff. The Azores Maritime Zone Commmand is one the five maritime zones command which are subordinate to the Naval Command.

[14] Rough sea conditions corresponds to a degree greater than 5 in the Douglas sea scale. The Douglas Sea Scale is a scale which measures the height of the waves and also measures the swell of the sea using a scale that ranges from "0" to "9".

(FIR) that match the two SRRs respectively. The aerial SAR service is intended to respond to aircraft related accidents or emergency situations. Both systems (the SNBSM and SNBSA) work in close cooperation, but for accidents involving ships or persons in the water, it is the Portuguese Navy who coordinates all efforts through their MRCCs. This may imply requesting aerial units to the Portuguese Air Force, specially rotary wings aircrafts, such as helicopters.

Figure 9. Aerial SAR units used by FAP. From left to rigth: EH-101 Merlin and EADS C-295M
Source: ©Portuguese Air Force

In maritime SAR operations, helicopters are usually used for retrieving persons from the water or from vessels. Fixed wing aircraft, such as the EADS C295[15], are usually employed for search operations. Recently, a fixed wing aircraft from the Portuguese Air Force, during a surveillance mission for Frontex[16], dropped a liferaft into the sea and rescued 34 migrants who jumped from a black dinghy after flames erupted near the craft's outboard motor (see [72] and [73]). This type of procedure can be a valid action in a SAR operation if the weather conditions are adequate. Dropping liferafts with specialist personnel to recover survivors is an unorthodox solution to be used in a mass rescue operation that still requires appropriate doctrine to be put into practice. Portable liferafts that can be thrown from an aircraft are allready being manufactured (see [74]).

Due to the large dimensions of the SRRs (particularly the SRR of Santa Maria in Azores), there are zones within them in which a naval SRU can take more than 4 days to reach. For example, the southwest corner of Santa Maria SRR distances over 1400 Nm

[15] The Portuguese Air Force operates twelve EADS C295 in two variants: the C295M (military transport version) and the C295MPA (maritime patrol aircraft version).

[16] Frontex is the shortname for "European Border and Coast Guard Agency" which was established in 2016 by Regulation (EU) 2016/1624 of the European Parliament and of the Council. It replaced the "European Agency for the Management of Operational Cooperation at the External Borders of the Member States of the European Union" who was also known by the same shortname. The mission of Frontex is to promote, coordinate and develop European border management in line with the EU fundamental rights charter and the concept of Integrated Border Management.

from Terceira Island in the Azores archipelago. Assuming a constant speed of 15 kts, a naval vessel, takes 13 hours and 20 minutes to reach the limit of the Portuguese EEZ, assuming he starts is voyage near the MRCC, and almost 4 days to reach the southwest corner of the Santa Maria SRR.

Coping with long distances between facilities and possible incident locations within the SRRs, it is relevant to depend on other types of tools that can help to shorten the distance between those who require assistance and those who can provide it. One of these "tools" are the tracking systems used by vessels to report their position and other relevant voyage data. The Automatic Identification System (AIS)[17] is one of the most common tracking system used by vessels around the world that enables communications between ships to ships and ships to coastal stations. For more information regarding the AIS system see the IMO resolution MSC.74(69) [28] (which defines the AIS system performance standards), the ITU recommendation ITU-R 232/8 [75] (specifies the AIS technical characteristics) and the IALA guidelines on AIS (see [76] and [77]). The signal sent by AIS transceivers can also be detected by satellites and this has enabled agencies to cover portions of sea very far away from coastal stations. Another type of tracking system that provides information about the identification and position of fishing vessels is the Vessel Monitoring System (VMS). The portuguese VMS, created in 1987, designated by MONICAP[18], was the first to be implemented in Europe, being presently operated in fishing vessels with length overall higher than 15 meters for fisheries control purposes. The Portuguese Navy receives MONICAP data sent by the Portuguese Fisheries Directorate[19] on a daily basis. The VMS data received includes all equipped fishing vessels inside the portuguese EEZ and all the portuguese fishing vessels around the world. This data is collected to a data base and is used to co-ordinate fisheries control missions and also to plan search and rescue operations. One of the drawbacks regarding the use of this data in search and rescue operations is the VMS data sharing policy among States. National authorities only have

[17] Automatic Identification System (AIS) is an autonomous and continuous broadcast system, operating in the VHF maritime mobile band. It exchanges information such as vessel identification, position, course, speed, and other relevant data between participating vessels and shore stations. It can handle multiple reports at rapid update rates, using Time Division Multiple Access (TDMA) technology to meet these high broadcast rates. Chapter V of the 1974 SOLAS Convention (as amended) requires mandatory carriage of Automatic Identification System (AIS) equipment on all vessels constructed on or after 01 July 2002.

[18] MONICAP is a monitoring system for the fishing activity inspection that uses Global Positioning System (GPS) for the vessel location and Inmarsat-C technology for the satellite communications between ships and a terrestrial control centre.

[19] The Portuguese Fisheries Directorate (DGRM is the shortname for "Direcção -Geral de Recursos Naturais, Segurança e Serviços Marítimos"), created by the decree-law n.º49-A/2012 of 29 February, is a central service within the direct administration of the Portuguse State with administrative autonomy. DGRM primary mission is to regulate and control the fishery activity and guarantee its compliance with the fishery policy.

access to the VMS data inside of their own EEZ. In the Portuguese case, authorities can only view foreign fishing vessels if they are inside the portuguese EEZ. The article from Kalyvas et al. [78] examines geospatial free-off charge data sources and discusses the various classes of available data and how these are used by maritime information systems.

The Portuguese Navy uses several maritime informations systems[20] to monitor vessels equipped with AIS and VMS devices. One of these systems is the Oversee information system, developed by Critical Software [79, 80], for maritime situational awareness. Oversee is an integrated solution for maritime operations centres designed to support coast guard functions that combines different data sources (AIS, VMS, radar, metoc data, etc) with analysis functionalities in a web based collaborative environment. More information about the Oversee solution can be found in [81]. Presently, Oversee is being operated in both MRCCs and it allows MRCC operators to manage a SAR case during its development. Using the maritime picture display (see Figure 10) and different types of GIS tools, the operator can calculate search areas (using drift models and metoc data) from the last known position (and time) of an object and obtain the best search pattern. It also allows to easily identify the nearest ship's to a certain incident location and assess the fastest ship to reach the scene.

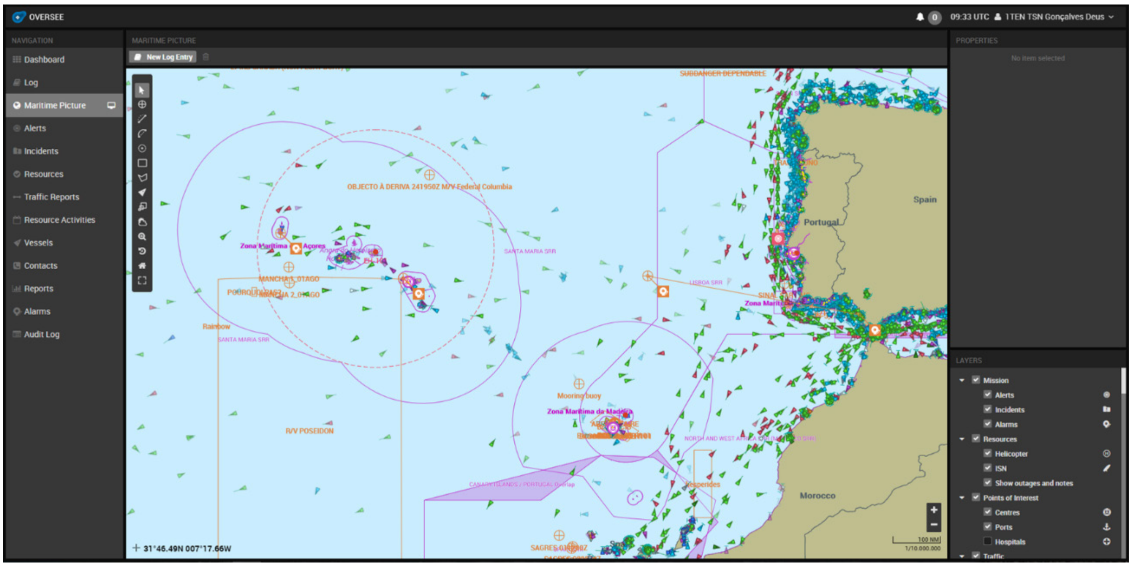

Figure 10. Oversee's maritime picture showing positions of vessels equipped with AIS and MONICAP
Source: Portuguese Navy (Naval Command)

Other maritime information systems used by the Portuguese Navy for SAR purposes is the SafeSeaNet solution from the European Maritime Safety Agency (EMSA) [82].

[20] A maritime information system is a geographic information system (GIS) designed to capture, store, integrate, manipulate, analyze, manage, and visualize all classes of maritime geospatial data, which are capabilities serving a cross-section of disciplines.

Alongside the maritime information systems, the Portuguese Navy, through the Portuguese Naval School Research Centre (CINAV) [83] and the Superintendency of Information Technology, stores AIS and VMS data for analysis purposes (see [84], [85], [86], [87] and [88]). The analysis of large quantities of AIS and VMS data is intended for mapping shipping densities and routes [89], creating automatic alerts for situational awareness [90], ship grounding and collision risk assessment [91] and to support search and rescue activities [92]. The Portuguese Navy has developed a prototype to visualize and analyse AIS and VMS data called "AISINTEL" which supports the studies previously cited.

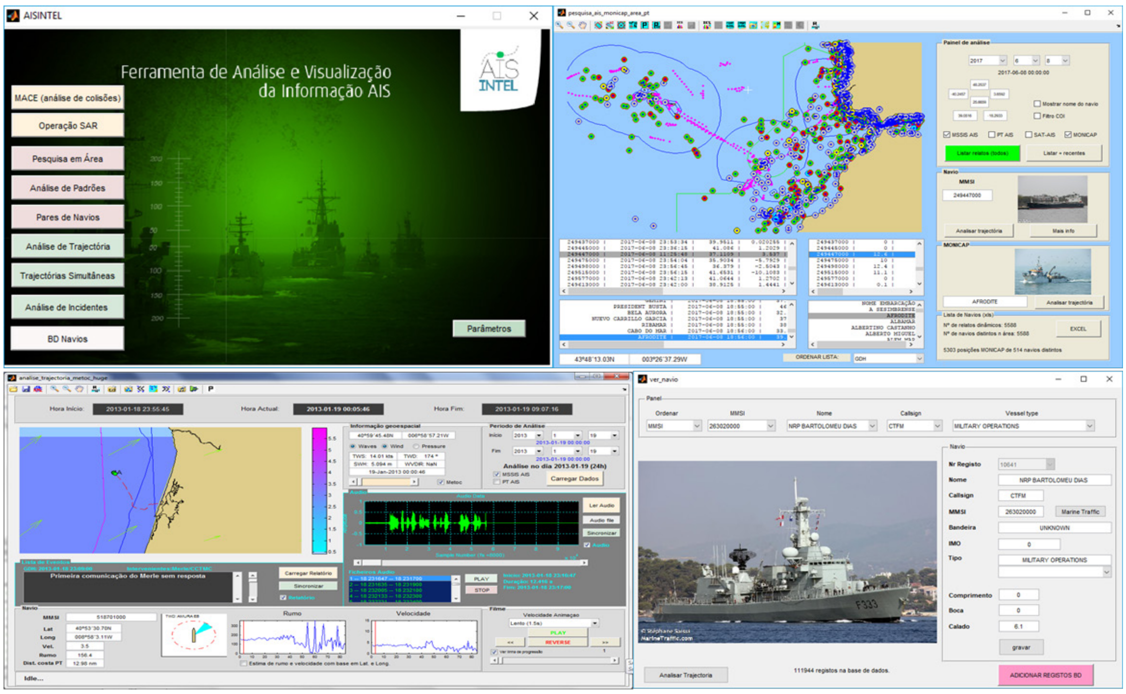

Figure 11. AISINTEL prototype for AIS and VMS data analysis

Source: Portuguese Navy (Naval Command)

This prototype (see Figure 11) is implemented using MATLAB technical language and is used by the Portuguese Naval School students to test and implement new procedures and tools to analyse geospatial data.

The analysis of AIS data has been pursued by an increasing large number of researchers around the world to study knew arising problems related with maritime traffic. Examples of AIS data analysis for maritime anomaly detection can be found in [93], [94], [95] and [96]. Studies related with environmental pollution caused by vessel's gas emissions to the atmosphere can be found in [97], [98], [99] and [100]. Studies related with maritime spatial planning (MSP) and mapping shipping densities can be found in [101], [102], [103] and [104].

Recently, the Portuguese Navy has recognised the need to map the cruise ship density and routes in order to identify areas remote from SAR facilities[21]. This concern is based on the increasing number of passengers that cross the national SRR and the Portuguese Navy's desire to ensure their safety. This work is being carry out by the Portuguese Naval School Research Centre and it comprises the analysis of large quantities of AIS data to map areas of low shipping density used by large passenger ships during their transits within the Portuguese SRRs [105]. Some of the preliminary results from this work are used in Chapter 4 to demonstrate the MMRO model with real data from passenger vessels and nearby ships.

2.1.5 Risks and threats for cruise ships

The history of maritime transport is marked by ship accidents with partly disastrous consequences on human lives and impact on society and the marine environment. In response to these disastrous accidents, more and more new requirements and amendments of existing regulations for the safe maritime transport were introduced by the International Maritime Organization (IMO). In [106] are summarized the major amendments made to the SOLAS Convention regarding passenger ships safety. Due to the high number of passengers carried by cruise ships[22] it is expected that authorities and specialized agencies show interest and concern in evaluating the risks and threats with this type of ships. In 2007, the USCG completed a Mass Rescue Operations Scoping Study (MROSS) that identified the largest potential response gaps were associated with USCG response to significant numbers of survivors from a passenger vessel. The two most likely scenarios were "Domestic passenger vessel requires evacuation" and "Large vessel sinks, passengers and crew must be located and rescued". The final report [107] made by the Research & Development Center (RDC) of the USCG, which was built upon the MROSS results, validated the most likely mass rescue incident scenarios and pointed several recommendations regarding equipment acquisition and response gaps to MROs.

The Cruise Lines International Association (CLIA), which is the largest cruise ship association, estimated that 24 millions passengers were carried worldwide in 2016 [108] and projects over than 25 million passengers will be sailing worldwide in 2017. The Equasis

[21] The need to identify areas remote from SAR facilities, particularly those areas that have passenger ships routes, within the Portuguese SRRs was recognized by the Naval Command in 2017 during an IT inspection. The following recommendation suggests that the problem should be addressed by the Portuguese Naval School Research Centre (CINAV) and the Directorate of Information Analysis and Management (DAGI) of the Superintendence of Information Technology since it will require the analysis of large quantities of AIS data.

[22] The SOLAS Convention defines "passenger ship" as a ship that carries more than 12 passengers (a "passenger" is any person other than the captain, crew or any person involved in the ship's business). A cruise ship is a passenger ship that carries people on voyages for pleasure, typically calling in at several places.

database [109], in its 2015 merchant fleet report, states that the passenger worldfleet is totalled at 6.741 vessels, which accounts for 7.7% of the total number of vessels worldwide. From the 6.741 vessels, 465 passengers ships displace more than 25.000 GT (Gross Tonnage) carrying on average more than 1000 persons (see table 1 in [110]). Currently, the largest passenger ship in the world, 'Harmony of the Seas' [111], entered service in 2016 and carries more than 5.000 passengers onboard and is operated by 2.300 crewman. CLIA expects 26 new cruise ships for 2017, which half are ocean liners (see slide 8 in [112]). In [113] is described the economic impact of the cruise industry in the world. Despite the cruise industry being a global industry, it still remains quite geographically and economically concentrated in North America and Europe.

In recent years, several high profile disasters with cruise lines took place and received great attention by the media. The 2012 Costa Concordia disaster and subsequent event with Carnival Triumph[23] [114] brought the theme of cruise line safety under the spotlights. In Europe, over the period 2011-2015, half of the accidents with ships were of a navigational nature, such as contacts[24], grounding/stranding or collision and 24% of the accidents occurred with passenger ships [115]. Vairo et al. [116] present a survey on sea accident risk and cite several studies focused on statistics about accident frequency (see also [117], [118] and [119]). Goerlandt and Montewka [120] presents a review and analysis of risk definitions, perspectives and scientific approaches to risk analysis found in the maritime transportation application area. Focusing on cruise ships accidents, Talley et al. [121] study the determinants of property damage and injury severity from cruise vessel accidents. They find that human error is the main determining cause of accidents rather than environmental and vessel-related causes. In this study the data only included events investigated by the USCG (events from Europe or Asia are not included). More recently, Mileski et al. [122], evaluate the nature of mishaps in the current cruise industry by evaluating 580 mishap incidents from 1989 to 2013 through a two-stage measurement design. They propose a categorization of the cause of incidents into seven categories (lack of proper maintenance, human error by crew, flaw in ship design, unknown, the combination of ship design flaws and the lack of proper maintenance, the combination of human error by the crew and the lack of proper maintenance, and the combination of ship design and human error) and find

[23] Carnival Triumph is a 100.000 GT cruise liner that carries more than 3000 passengers and its operated by Carnival Cruise Lines. On February 10, 2013, the ship suffered a fire in the aft engine room. Although the fire was automatically extinguished and there were no injuries to passengers or crew, it resulted in a loss of power and propulsion. The incident left 3143 passengers adrift in the Gulf of Mexico for days.

[24] EMSA defines "contact" as an incident caused by ships striking or being struck by an external object. The objects can be: floating object (cargo, ice, other or unknown); fixed object, but not the sea bottom; or flying object.

that the main cause of accident is due to lack of proper maintenance, followed by human error.

Despite the causes involved in cruise ships accidents, the number of fatalities is very small compared with the total number of passengers that take cruise ships. Before the Costa Concordia accident, there were only 16 fatalities out of more than 100 million passengers that took cruise ships between 2005 and 2012 [123]. These numbers show that cruise ship safety is quite good. The same cannot be said regarding passenger ferries: the 2015 report on ferry accidents [124] reports 160 ferry accidents in the 14-year period between 2000 and 2014 which caused more than 16.000 fatalities worldwide (see relevant statistics in appendix of [124]).

The possibility of a terrorist attack to large passenger ships has become a serious security concern among intelligence analysts, law enforcement officials, and policymakers worldwide. Since the terrorist attacks of September 11, 2001, port security and the theme of maritime transportation has emerged as a significant part of the overall debate on U.S. homeland security [125]. Terrorist attacks in the Southeast Asia, in particular the Abu Sayyaf attack on a ferry in 2004 [126], have also contributed to the emerging concern of the vulnerability of maritime transportation to this new threat [127]. These studies have the common argument that the risk is partially influenced by the attractiveness of maritime targets, whether they refer to vessels or port facilities. The possibility of using a cargo ship as a floating bomb is also well documented [128]. In [129] the author defends that this last possibility is less likely when compared to land attacks. The RAND corporation report [130] was the first study to assess the risk (through the assessment of threats, vulnerabilities and consequences) concerning terrorist attacks on passenger vessels and containerized shipping based on a qualitative risk analysis procedure (see the appendix in [130]). In this report, Greenberg presents six scenarios of potential maritime terrorist attacks to cruise ships and ferries [130, p. 74]:

- Hijacking of a cruise ship and its passenger.
- Sinking a ship using a boat-borne IED.
- Sinking a ship with a parasitic device.
- Bombing on board a ship.
- Standoff attack on a ship using heavy artillery.
- Biological attack on a ship’s food or water supply.

Risk estimates were generated using qualitative methods that involved the use of defined ordinal scales to assess terrorists’ intents and capabilities, target vulnerabilities and attack consequences. The majors findings showed that an on-board bomb, an IED attack or food/water contamination were the attack methods considered to have the highest threat risk. Piracy and the risk of hijack were considered a much lower risk. Bowen et al. [131] examines the terrorist attack scenarios proposed by Greenberg et al. and incorporated them in a survey designed to estimate the costumer perception of safety and security on-board cruise ships. Bowen et al. found that the greatest risk was thought to come from a terrorist attack on a ship or a port by an extremist group (other risk options where political instability, wheather conditions, onboard illness, pirate attack on cruise ship, natural disaster, other) with a low level of risk. Asal and Hastings [132] argue about the reasons that cause terrorists organizations to attack maritime targets. In their study, they used the GTD database[25] to analyse terrorist events categorized by “maritime” in the “target type” variable. According to the GTD, between 1970 and 2010, there were 181 attacks where maritime facilities or civilian ships were the primary targets. Several factors that may be related to maritime terrorist attacks were studied (overall motivations of terrorist groups, ideology, capability, group size,etc). Group size, drug trafficking, control of territory, and network connections (variables related with the organization’s “capability”) were found to be positively associated with the lethality of terrorist attacks while the terrorist groups ideology and to a large extent their goals seemingly has little to do with whether they go to sea. In the end, capability seems to be the primary reason for terrorist organizations to purse maritime terrorism.

The last successful terrorist attack on a cruise ship (passenger ferries not included) was in 1985 when the Italian vessel Achille Lauro was hijacked by a group of terrorists from the Palestine Liberation Organisation. Since then, several other attempts have been tried by pirates without much success. However, the concern about such a possibility and its consequences are real and has received an increased interest by the academic community.

2.2 Related vehicle routing models and variants

The maritime mass rescue operations (MMRO) problem presented in Chapter 1 is an extension of the vehicle routing problem (VRP) with a special structure due to the vehicle’s feasibility of moving between time-indexed nodes. More specifically, the MMRO problem can be viewed as a multiple-depot generalized vehicle routing problem with profits defined over a (huge) layered graph discretized by time where arcs between time-indexed nodes are feasible only if vehicles are allowed to travel within the given time ranges. The multiple

[25] The Global Terrorism Database (GTD) can be found at the University of Maryland website at: http://start.umd.edu.

depot VRP, the GVRP and the Prize collecting VRP (PCVRP) are all extensions of the well known VRP and share common features with the MMRO problem.

The VRP is one of the most studied combinatorial optimization problems and one that has a huge impact in Logistics and transportation optimization. The interest the VRP has gained by the academic community has led to the research of several variants of the VRP model, even outside the logistic world (see for example applications using UAV in security context [133] [134], [135]). The VRP was first introduced in 1959, by Dantzig and Ramser [136], where a set of service stations (costumers) have to be supplied by a certain terminal (depot) using a fleet of trucks with equal capacity. The objective of the problem is to find the optimal set of routes that minimize the overall travelled distance. This problem is considered the "classical VRP" among the academic community and corresponds to the *capacitated vehicle routing problem* (CVRP). Later in 1964, Clark and Wright introduced a variant of the previous VRP model where vehicles had different capacities along with the well known *Clark & Wright savings heuristics* [137]. This version of the VRP came to be know as the *heterogenous fleet* vehicle routing problem or *mixed fleet* vehicle routing problem [138]. Several variants of the VRP soon appeared following the papers of Dantzig and Ramser and Clark and Wrigth and VRP has become one of the most studied topics in Operations Research. These variants included service time windows, maximum route length, multiple depots, pickup and delivery, backhauls, etc. Survey reviews focused on the problem definition and solution methods (dividing them into exact algorithms, heuristics and meta-heuristics) of these variants can be found in the books of Toth and Vigo [139] and Golden et al. [140]. Spurred by the complexities of real-world problems and processing capability of current computers, a large and increasing number of VRP variants have been studied in the last decades. This variability of problem characteristics and assumptions has led to some attempts to classify the VRP variants through a taxonomic approach. The articles of Eksioglu et al. [141] and Braekers et al. [142] present a taxonomic review of the VRP literature published since 1954. Eksioglu et al. reviews 1021 journal articles with "VRP" as the main topic and verifies that literature growth (cumulative growth) is almost perfectly exponential with a 6.09% annual growth rate between 1955 and 2005. Braekers et al. propose an evolved taxonomy for the VRP based on the latter by Eksioglu et al. which completes the review of the VRP literature between 2009 and 2015. Results indicate that VRP variants that include real-life characteristics and assumptions have received more attention from researchers than other related research topics. Another finding in [142] is that many researchers propose highly problem-tailored solution methods which are not directly applicable to other problem variants. Pursuing the development of such general solution approaches presents itself as a highly worthwhile endevour.

The next three subsections present a brief review of the VRP variants to which the MMRO problem is related and also the intrinsic structure based on a layered graph to cope with the time-dependent relation between nodes of different clusters. The last subsection relates the MMRO problem with the VRP variants.

2.2.1 Generalized vehicle routing problem and variants

The MMRO problem assumes that the location of drifting objects is known and a time-dependent prize function (normally an utility or profit function based on survival times) is associated with each object to be collected or retrived from the water. The time interval between the received alert (initial instant) and the last instant (that defines the mission duration where the SAR system is assessed) is discretized by a *time step* parameter (usually in minutes). The time step is important in setting the time stamp of the position of each object during its drift. The set of positions for each object is a cluster of nodes in the problem's graph representation. We shall assume that an object, independently of the number of persons it represents, requires to be visited (or collected) only once. This means that a cluster can only be visited once. As expected, the MMRO problem can be conceived as a generalized vehicle routing problem as the one introduced by Ghiani and Improta [3].

The GVRP is formally defined as follows: let $G = (V, A)$ be a directed graph where $V = \{0,1,2, \dots, n\}$ is the set of nodes or vertices and $A = \{(v_i, v_j): v_i, v_j \in V, i \neq j\}$ is the set of arcs. A nonnegative cost c_{ij} is associated with each arc (v_i, v_j). Node set V is partitioned into $k + 1$ nonempty subsets (or clusters) $V_0, V_1, V_2, \dots, V_k$, where $V_0 = \{0\}$ is the depot (the depot is node 0) and each node belonging to a cluster $V_i, i = 1, \dots, k$ has the same demand d_i (total demand of each cluster can be satisfied by any of its nodes). A fleet of m heterogeneous vehicles are available with equal capacity Q. The GVRP consists in finding minimum total cost tours of m vehicles starting and ending at the depot, such that each cluster is visited by exactly one vehicle at any of its nodes and the sum of the demands of each tour does not exceed Q. An illustrative scheme of the GVRP and a feasible solution with two vehicle tours is shown in Figure 12.

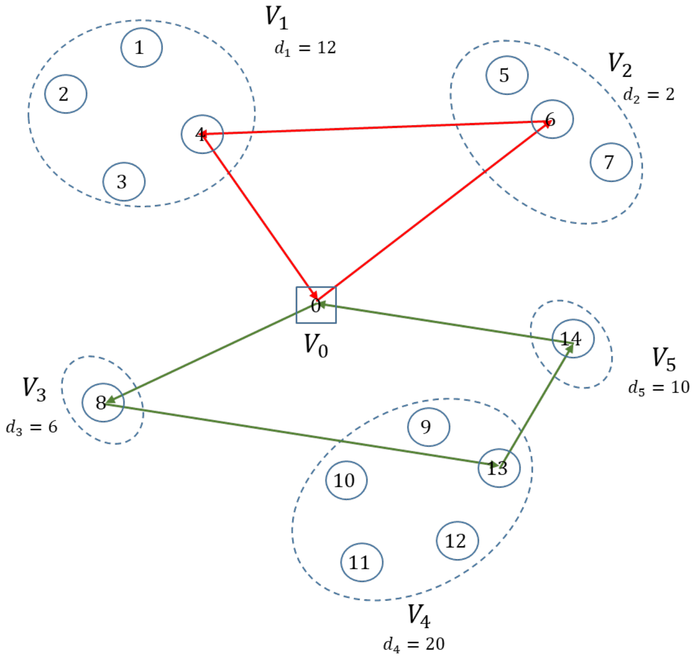

Figure 12. A feasible solution to the Generalized Vehicle Routing Problem

This problem was firstly introduced in 2000 by Ghiana and Improta [3] along with a transformation of the GVRP in to a capacitated arc routing problem (CARP) for which an exact algorithm and several approximate procedures are reported in literature. In their article, they refer the post-box collection problem described in Laporte et al. [143] as a real-world situation that can be modelled as a GVRP if more than one vehicle is required. They also mention the possibility to model the distribuition of goods using a fleet of vessels that have to supply costumers in islands by visiting only one of their harbour. The article of Baldacci et al. [144] presents several examples of applications where the GVRP model can be employed: the travelling salesman problem with profits can be modelled as a GVRP; several extensions of the VRP, namely the *VRP with Selective Backhauls*, the *Covering VRP*, the *Periodic VRP* and the *Multi-Depot VRP* can also be modelled as a GVRP; and the *Capacitated General Windy Routing Problem* (CGWRP) (see [145] where a variant of the CGWRP with turn penalties is modelled as a GVRP) is also shown how to be modelled as a GVRP. The last given example of the GVRP application consists in the design of tandem configurations for Automated Guided Vehicles (AGV) which have to load, move and unload materials around a manufacturing facility or warehouse.

The problem at hand in this thesis stands as another example where GVRP models can be used in a real-world problem. In this specific case, the GVRP model is used for assessing the efficacy of the SAR response to a large-scale maritime incident. A real-world application of the model proposed in Chapter 4 consists in a study where several scenarios are designed by SAR experts concerning specific maritime areas and different degrees of severity envolving large scale accidents with cruise ships in order to assess the efficacy of the SAR system response. This study would consider different resources availability and location and would provide a sensitivity analysis regarding the efficacy of the response when more resources are available or are at different locations. The results from such study would provide rational arguments to sustain strategical alternatives regarding the acquisition of new SAR resources. The main idea with such study is to identify the capability gaps, as to resources availability, in the SAR system if a MRO would be required. In this sense, the MMRO problem can be understood as a special case of a *Disaster Operations Management* (DOM) problem. Altay and Green [146] review the literature on disaster operations management and group the activities of disaster operations management into four phases: mitigation, preparedness, response, and recovery. Caunhye et al. [147] categorize disaster operations between predisaster operations (short-notice evacuation, stock prepositioning, facility location for shelters, stores, and medical centers) and postdisaster activities (relief distribution, casualty transportation). In a more recent paper, Balcik [148] defines the Selective Assessment Routing Problem (SARP) which is formulated as a variant of the team orienteering problem (TOP) with a coverage objective. The purpose is to quickly evaluate the impact of a disaster on community groups within an affected region for estimating the need regarding humanitarian help. Sites may carry multiple characteristics (i.e., coastal and high impact) and the coverage objective is related with the number of "critical characteristics" observed by the teams. This assessment is made by selecting a number of sites in an affected region that must be visited by teams. A 3-index integer linear formulation is proposed for selecting the sites and routing the available teams. The structure of the SARP has similarities with the GVRP since the set of nodes carrying a particular critical characteristic can be considered as a cluster. In the SARP more than one node in each cluster can be visited in order to achieve the desired coveraged.

The MMRO problem arises naturally in the response phase in a postdisaster situation. Nonetheless, using simulation and scenario analysis, the MMRO problem can be used for preparedness or predisaster operations. The relevant feature of the MMRO problem stands in the GVRP variant that has to be solved in order to quantitfy the expected efficacy of the SAR response to a large-scale maritime accident.

The latest research on the GVRP ranges from examining integer linear programming formulations and exact methods based on branch and cut algorithms (see [4], [149], [150], [151]), heuristics and metaheuristics methods (see [152], [153], [154]) and some transformations of the GVRP into other variants of the VRP (see [3] and [149]). Practical applications of the GVRP model to solve real-world problems are laking in the literature.

Kara and Bektas [4] proposed the first integer linear programming (ILP) formulation of the GVRP and shows how this model reduces to the well-known generalized multiple travelling salesman problem (GmTSP), generalized travelling salesman problem (GTSP) and the capacitated vehicle routing problem (CVRP). The GVRP is $\mathcal{NP}\text{-}Hard$ as it contains the CVRP as a special case. The proposed ILP formulation is a flow based formulation that uses the well-known Miller-Tucker-Zemlin (MTZ) constraints for the TSP adapted for the CVRP with a polynomial number of variables and constraints. In 2011, both Bektas et al. [150] and Pop et al. (see [149], [155] and [156]) test different integer linear programming formulations for the GVRP. In their article, Bektas et al. [150] propose four different ILP formulations for the GVRP: two based on multicommodity flow and the other two based on exponential sets of inequalities. A branch-and-cut procedure based on these two last formulations and a large neighbourhood search (LNS) heuristic for the GVRP with limited fleet is also presented along with computational results.

As for heuristics methods, the first proposed method is due to Bautista et al. [157] which uses an ant colony heuristic to solve an urban waste collection problem based on a ILP formulation of a special case of the GVRP model. The waste collection problem is presented as a mixed capacitated routing problem with turn constraints (MCARPTC) which is transformed into a GVRP. In [152], Pop et al. also presents a metaheuristic based on a ant colony system (ACS) to solve the GVRP problem. Computational results for several benchmark problems are also presented. A genetic algorithm for the GVRP problem is proposed in Pop et al. [158] which outperforms the ACS heuristic in [152] for the same set of test problems. Moccia et al. [154] presented an incremental tabu search heuristic to solve a variant of the GVRP with time windows (GVRPTW). More recently, Navidadham et al. [159] propose a combination of parallel universes' algorithms [160] in addition to the Tabu search to solve the GVRP.

The cluster structure and the requirement to visit only one node in each cluster are the main features which the MMRO problem share with the GVRP. Due to the characteristics of SAR operations, namely the availability of different assets who are positioned in different locations, other features have to be considered, that will give rise to variants of the GVRP. One of theses variants is the multiple depot generalized vehicle routing problem (MDGVRP). In this variant of the GVRP, a set of depots are considered where at least one vehicle will start and end its tour from each depot. The MDGVRP extends naturally the multiple depot vehicle routing problem (MDVRP). In fact, the MDGVRP reduces to the MDVRP if each cluster is a singleton. In [10] Montoya-Torres et al. present a state-of-the-art survey on the vehicle routing problem with multiple depots. Most of the literature on the MDVRP consider that each vehicle start and end the tour at the same depot. In the MMRO problem we want to allow the starting node to be different from the ending node for some of the available vehicles. This feature is not very common in the literature. In the MMRO problem it could also be plausible to admit that for some vehicles their tour doesn't have to end in a depot. This is the case of some vessels that will resume their planned voyage after being released from the rescue operation by the MRCC. So, three situations can be considered regarding how the vehicle's tour can end in the MMRO problem:

1) the vehicles start and end their tour at the same depot;
2) the vehicle do not necessarily end their route at the same depot from where they start;
3) the vehicle does not end the tour at a depot.

The first situation correspond to the common formulation of the MDVRP. The second situation is much less common to find in the literature. In [161] Afshar-Nadjafi and Afshar-Nadjafi present a mixed-integer programming formulation for the time-dependent multi-depot vehicle routing problem where vehicles do not end their route at the same depot from where they start. The last situation correspond to the *open vehicle routing problem* (open VRP). A recent survey on the open VRP can be found in [162]. Very recently, Soto et al. [163] addressed the Multi-Depot Open Vehicle Routing Problem (MDOVRP) where vehicles start from different depots, visit customers, deliver goods and are not required to return to the depot at the end of their routes.

The binary linear programming formulation proposed for MMRO problem in Chapter 3 allows vehicles to end their tour at a specific set of depots which may not contain the starting depot. In this sense, the MMRO problem has some similarities with the problem in [161].

The prize collecting or profit feature of the MMRO problem comes from the possibility of not being able to retrieve all objects within the mission duration. The prize collecting feature has gained interest when linked to the traveling salesman problem, since it generalizes the TSP and has many applications in real-world problems. The traveling salesman problem with profits (TSPs with profits) is the term used when a single vehicle is involved. More general problems in which several vehicles might be involved are called routing problems with profits. In these kind of problems there are usually two opposite objectives, one pushing the salesman to travel (to collect profit) and the other inciting him to minimize travel costs. Feillet et al. [6] presents a survey on the TSP with profits where it covers different generic problems that make up this class of problems, main applications in real-world problems, formulations and structural properties, exact and heuristic and metaheuristic solution procedures and also mentions the single vehicle and multivehicle variants. In their survey Feillet et al. [6, p. 189] consider three generic problems that together make up TSPs with profits, depending on the way the two objectives are addressed:

1) One objective function that combines both goals: the aim is to minimize travel costs minus collected profit.

2) Travel cost is stated in the constraints and the aim is to maximize collected profit such that travel costs do not exceed a certain limit.

3) The profit is stated as a constraint and the aim is to minimize travel having a collected profit not smaller that a certain value.

The problems mentioned above have appeared under several names in the literature. The first problem has been defined as the profitable tour problem (PTP) by Dell'Amico et al. [164]. The second problem is known as the orienteering problem (OP) [165]. Other names under which the OP can be found are the selective TSP (see Laporte and Martello [166]) and the maximum collection problem (see Kataoka and Morito [167]). The third problem is known as the prize-collecting TSP (PCTSP) and was first introduced by Balas [5] in 1989. The prize-collecting TSP is also known as the quota TSP by Awerbuch et al. [168].

From the three mentioned problems, the MMRO problem is closer to the second problem than the others, which is the orienteering problem. Since the profit is related with the value of human life (although the human life is measured by the time spent in the water by a SAR object), it makes little sense to combine travelled distance with human life value. That would imply a comparison between two factors that simply should not be compared. This excludes the first type of problem which combines the distance and profit objectives. In the third problem we are considering a limit on the value of human life and that is ethically and morally not acceptable. This reason, alone, excludes the third approach of modelling profit.

Thus, the MMRO problem will have a profit stance stated in the objective function. Stating the travelled distance in the constraints is a very reasonable modelling option because for some types of vehicles, namelly helicopters, working beyond the operational range will put lifes at risk (at least the helicopters crew's life).

The vehicle routing problem features described in this subsection give a more detailed view of the MMRO problem and its vehicle routing structure. One aspect in the vehicle routing problem which is also present in the MMRO problem involves avoiding subcircuits in the solution tours that do not include the starting depot for each vehicle. This topic is described in the next subsection.

2.2.2 Layered graphs

One of the most relevant features of the MMRO problem stands with the time-dependency of the profit function associated with each object that we wish to retrive from the water. The time-dependency component does not influence solely the objective function of the problem. It also conditions the feasibility of vehicles when moving between time-indexed nodes. The graph definition of the MMRO problem presented in subsection 1.1.1 does not specify the time stamp index associated with each node. The feasibility of arcs between nodes of different clusters is implicitly coded on the subsets of arcs $A_k \subseteq A$ for each vehicle k. Since time will be discretized into time stamps with a given time step parameter, it is possible to obtain an equivalent graph where the nodes will be grouped not only by their respetive cluster but also by their respective time stamp index. Each time stamp index corresponds to a specific moment in time, usually represented using the date-time-group format[26]. Let $T = \{0,1,2,\dots,h\}$ be the set of time stamps indexes for which it is known the position of each object during its drift. The index 0 corresponds to the time instant where all vehicles leave their starting depot. Since costumers are far away from the starting depots, it is not necessary to consider in the graph the nodes of the costumers at time 0. One can describe the MMRO problem on a layered graph where the clusters and time stamps indexes are organized into layers. In this representation the nodes can be writen using the notation $(i,t), i \in I_S, t \in T$.

To illustrate this new representation of the MMRO problem, let us consider the example given in subsection 1.1.2 restricted to the first four SAR object (S_1, S_2, S_3, S_4) with a feasible solution involving two vehicles.

[26] The date time group (DTG) is a set of characters used to express the year, the month, the day of the month, the hour of the day, the minute of the hour, and the time zone. NATO members use the format DDHHMMSSZ MMM YY to describe a specific moment in time. For example, 201224Z SET 17 represents 1224 UTC in 20 september 2017.

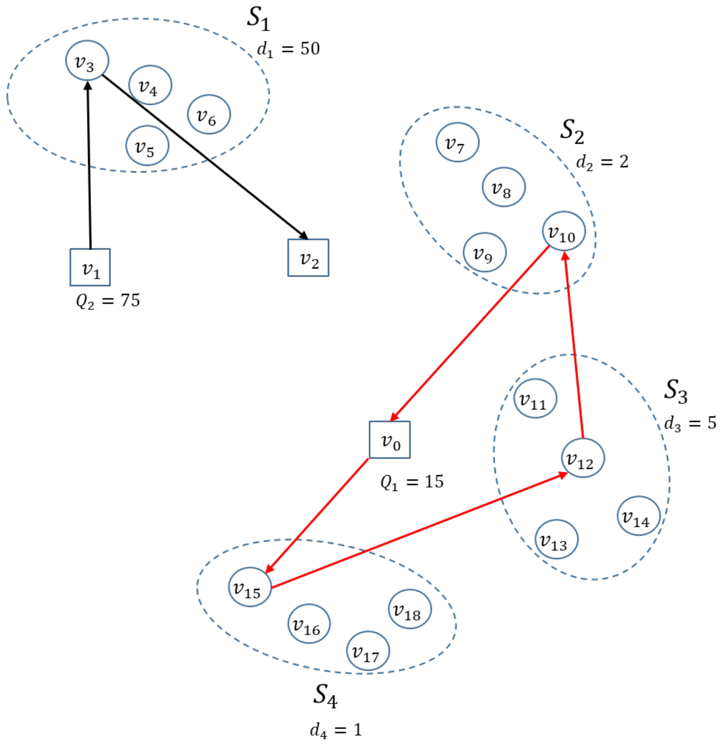

Figure 13. Illustrative example of MMRO solution in original graph

Using the cluster index and the time stamp index it is possible to rearrange the nodes into a cluster-time grid where a feasible solution is a set of paths, one for each vehicle, between their respective starting node and a feasible ending node, that visits at most only one node in each cluster layer. In the example shown in Figure 14 a copy of the node v_0, denoted by v'_0, is added to the set of nodes in order to guarantee that "ending" nodes will appear on the rigth side of the layered graph. In Figure 14 only the set of costumers is structured into cluster-time layers. The set of starting depots and the set of ending depots is not discretized in time. This implies that all the vehicles leave their depot at the same time, which is at time 0. The instant where vehicles terminate their tour at an ending depot is also not discretized in time. The vehicle's expected time of arrival from a given costumer node $(i, t), i \in I_S, t \in T$ to an ending node can be easilly calculated using the average vehicle speed and the distance between the costumer node and the ending depot.

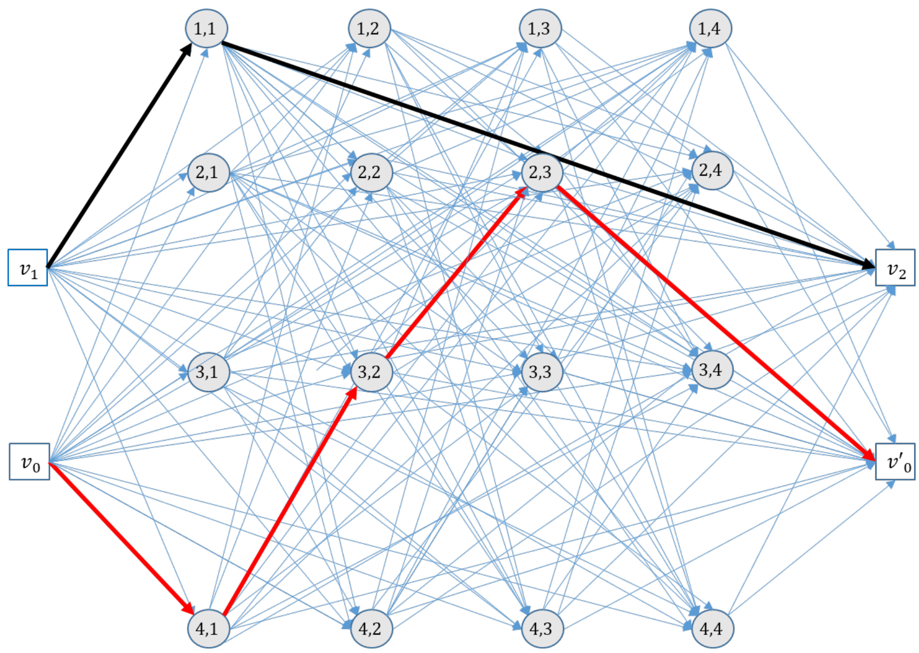

Figure 14. Layered graph representation of the MMRO instance

With the layered graph representation one can observe that there can be no arcs between nodes in differents clusters with the same time stamp index (that would imply that a vehicle would travel between two separate locations instantaneously) or arcs (i, j) connecting nodes $(w(i), t_i)$ and $(w(j), t_j)$ where $t_j < t_i$ (which otherwise would imply that a vehicle could travel back in time). Thus the arcs have only one direction in the time axis (no backward arcs, upward or downward arcs). Arcs between nodes within the same cluster are also not feasible due to the assumption that once a node is visited all the demand is satisfied, which means all survivor's are retrieved.

Assuming that vehicles travel with constant speed between time-indexed nodes and that vehicles do not delay their departure once a visited costumer is serviced (all survivors are retrieved), then it is reasonable to consider that for each costumer time-indexed node $(i, t), i \in I_S, t \in T$ there will be only one arc to each of the remaining clusters $S_j, j \in I_S \backslash \{i\}$. If the travel distance between any two costumer nodes in different clusters is one time unit, then if a vehicle visit node (i, t) at time index t then it can only move to nodes $(j, t+1), j \in I_S \backslash \{i\}$ or an ending depot. Off course, this is the most simple case where all vehicles have the same speed and there are no delays when "servicing" a costumer. In maritime operations, one can consider a unique average speed for all vessels, specially if they operate in ocean areas. But since we are considering helicopters operating simultaneously with vessels then more than one average speed must be considered. The number of arcs leaving a costumer time-indexed node $(i, t), i \in I_S, t \in T$ will depend on the number of different average velocities associated with the available vehicles. If vessels vehicles move with an average speed of 10 knots and helicopters move with an average of 100 knots, at least two arcs should connect a given costumer indexed node to a different costumer cluster. If a vessel moves at an average speed sp_1 and departures from a node $(i,t), i \in I_S, t \in T$ it will arrive at node (j,t') where the time index t' corresponds to the time stamp of time index t plus the travel time at speed sp_1. If a helicopter moves at an average speed $sp_2 > sp_1$ and departures from the same node $(i,t), i \in I_S, t \in T$ it will arrive at node (j,t'') where the time index t'' holds the relation $t'' < t'$. Thus we will have two distinct arcs leaving a node $(i,t), i \in I_S, t \in T$ if there are two types of average speed made by the available vehicles.

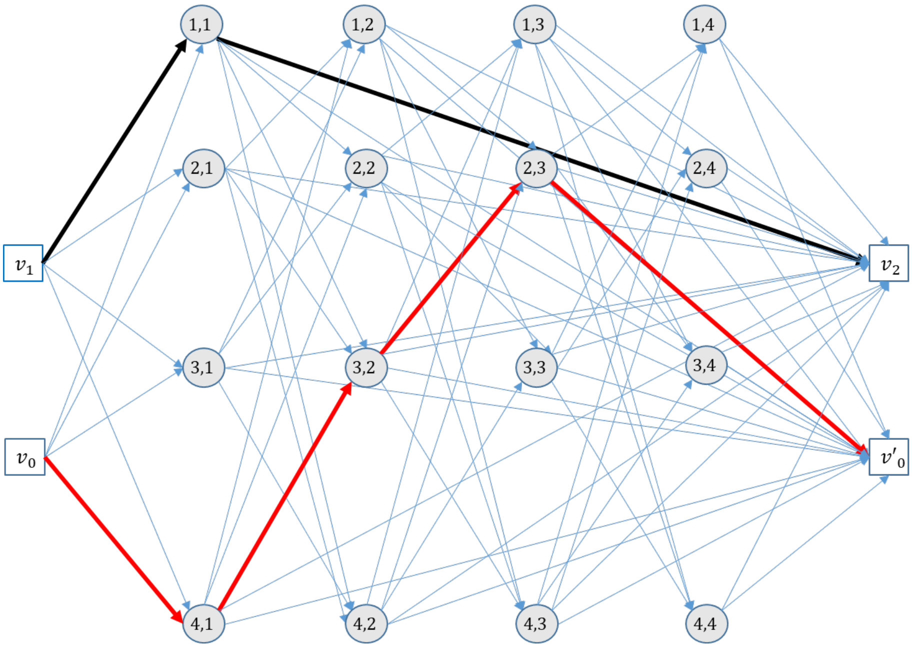

Figure 15. Layered graph with arcs associated to feasible movements of vehicles that move at the same speed

Figure 15 depicts the layered graph when all vehicles move at the same speed and movements between nodes of different clusters cost one time unit. It follows that only arcs between time index nodes where the time index difference is equal to one are feasible. The kinematics associated with the movement of vehicles through nodes that represent the location of objects through time will be made implicitly in the layered graph. This is one of the advantages of using the layered graph representation of the MMRO problem instead of trying to state those constraints into an integer linear programming formulation. If the layered graph complies with the vehicles kinematics when moving between time-indexed nodes within the expected time ranges, then a flow based formulation will not require additional constraints to avoid unfeasible subcircuits in the solution. This is another advantage that follows from developing a flow-based formulation from the layered graph structure. The construction of instances of the MMRO problem and the creation of feasible arcs between time-indexed nodes for vehicles with different speeds are detailed in subsection

3.1.3. An arc-based formulation based on the layered graph representation of the MMRO problem is described in Section 3.2.

The concept of layered graph in network design problems is usually associated to formulations and has gained widespread attention in the recent years. New formulations for several network design problems based on layered graphs have been proposed recently which show computational advantage over previous ones. To the best of our knowledge, the first reference to a formulation based on a layered graph is due to Picard and Queyranne [169] who proposed an integer linear programming formulation based on a multipartite graph for the time-dependent travelling salesman problem (TDTSP). They presented a branch-and-bound algorithm based on this formulation to minimize the tardiness cost in one-machine schedulling problem. The formulations for the TDTSP based on layered graphs proposed by Picard and Queyranne have been widely studied and several families of valid inequalities have been proposed to be used in branch-and-cut algorithms (see [170], [171], [172], [173]). Another early reference to layered graphs can be made by interpreting the network flow formulation by Steward [174] for the problem of optimal allocation of search effort. In this problem the purpose is to find an optimal allocation of search effort (effort can be measured by the time a sensor is searching for a target in a specific region or area) considering that a target moves between a set of cells during a finite set of periods of time according to a specified Markov process. The target path through time is given by a probability vector and if the sensor and target meet in the same cell then the probability of detection, given that the target is not detected earlier, is an exponential detection function similar to the one considered in Stone [32]. Eagle and Yee [175] propose a branch and bound algorithm for this problem, with the bound calculated by solving a relaxed problem using the Frank-Wolfe method [176]. Improvements on the Eagle and Yee procedure for this problem were made by Martins [177] where an improved bounding procedure was based on the solution of a single longest path problem that maximizes the expected number of detections.

In 2009, Gouveia et al. [178] show that the hop-constrained minimum spanning tree problem (HMSTP) is equivalent to the Steiner tree problem (STP) in an appropriate layered graph and provides computational results in which the direct cut model for the STP defined in the layered graph performs best than previously known models for the HMSTP. Ljubić and Gollowitzer [179] followed the layered graph idea of Gouveia et al. and propose three formulations based on layered graphs to model the Hop Constrained (HC) Connected Facility Location problem (ConFL) as a ConFL. In [180], Sinnl and Ljubić present a node-based model for the Steiner tree problem with revenues, budget and hop-constraints (STPRBH) where arc variables of the problem's layered graph are projected and the new model relies only on variables associated to the nodes of the layered graph. Thus, the resulting ILP formulation is considerably smaller which allows to solve instances based on larger graphs and hop-limits. More recently, Gouveia et al. [181] study a novel approach to solve the black-and-white traveling salesman problem (BWTSP) based on several variants of position-and-distance-dependent reformulations, each of these with its own associated layered graph.

Formulations based on layered graphs have been studied as an alternative approach to previous methods to solve many combinatorial problems, including several variants of the TSP and also several variants of the hop-and-diameter constrained spanning trees. In the MMRO problem the layered graph is used to satisfy the feasibility of the vehicles movements between the objects location through time. This is done implicity during the creation of an MMRO instance (see subsection 3.1.4) and leads to an integer linear formulation that does not require additional constraints to avoid unfeasible subcircuits in the vehicles tours.

2.2.3 Reduction of the MMRO problem to various routing problems

This subsection describes how the MMRO problem relates with other VRP models. The MMRO problem is a special case of the GVRP which has a special structure due to the vehicle's feasibility when moving between time-indexed nodes. The additional constraints that state which arcs can be used by each vehicle are implicity defined in a layered graph. If no such constraints were to be imposed on the problem and only one depot is available then the MMRO problem would be a natural extension of the GVRP and the VRP. Considering the MMRO problem definition presented in subsection 1.1.1, the MMRO reduces naturally to the GVRP when the following conditions are met:

- There is only one depot.
- No distance limit on the length of the vehicles tour ($L_k = \infty, k = 1, \dots, K$)
- No arc feasibility constraints associated to vehicles, which means the sets A_k are equal to the set of arcs A for all vehicles.

It is important to note that the MMRO problem has a time-dependent objective function which can be explicitly specified in the cost matrix C since each node is time-indexed. The MMRO problem reduces to the CVRP when the next following conditions are added to the previous ones:

- Clusters have exactly one node.
- Capacity is equal for all vehicles, $Q_k = Q, k = 1, \dots K$.

- Objective function is not time dependent.

In the context of maritime search and rescue operations, one can consider the problem of retrieving a set of objects from the water as soon as possible when there is no maritime drift acting on the objects. In such a case, the routing problem does not require a set of nodes to represent the object movement in the water caused by the maritime drift (since the objects are considered to be static) and each cluster can have only one node.

Figure 16 is an adaptation of the scheme proposed by Kara and Bektas [4] where the GVRP model is reduced to several routing problems:

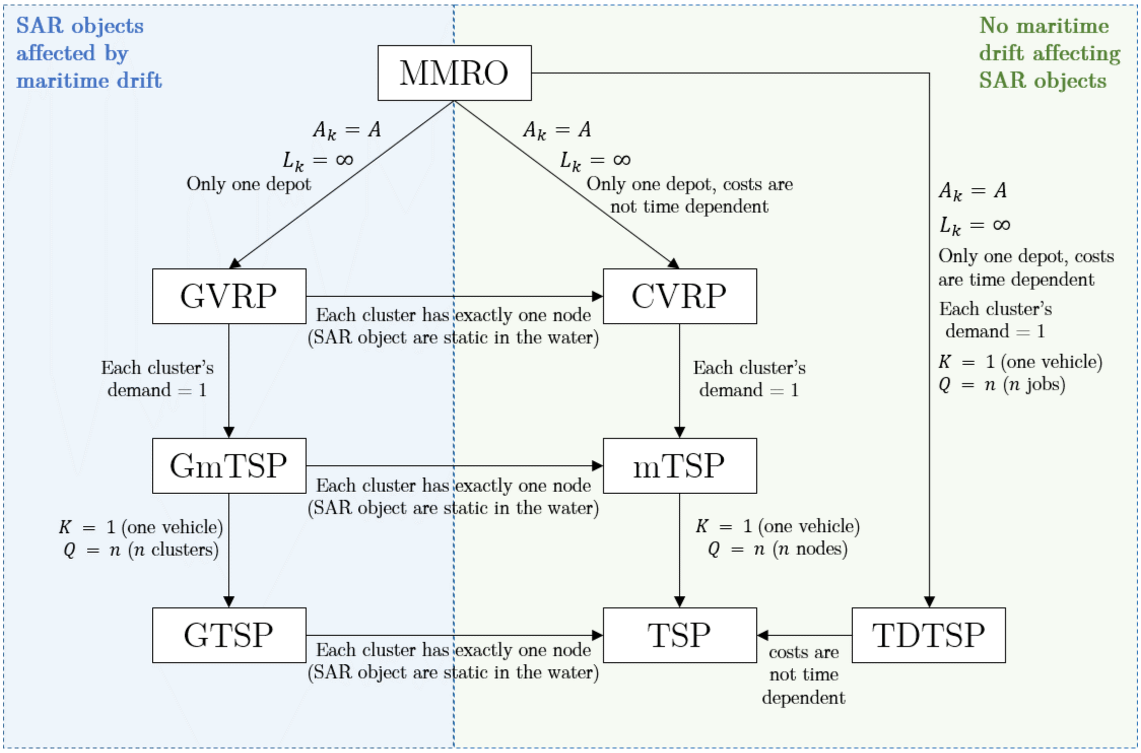

Figure 16. Reduction of the MMRO model to various routing problems

If we drop the assumption of time-dependent costs and assume that only one vehicle is available to retrieve dispersed persons in the water (costumer demand is one unit) which we also assume to be static (no maritime drift), then the problem becomes a TSP. In this later case, if we assume that the costs are time dependent, then the MMRO becomes a TDTSP. Figure 16 presents a distinct separation between routing problems that have (blue area) or do not have the "generalized" feature (green area). In the MMRO problem, the "generalized" feature is used to model the movement of objects caused by the maritime drift which in turn will affect the cost (or benefit) of retrieving them. If there are no drift forces acting upon the objects, then two situations have to be considered: the cost of retrieving an object is time dependent or not. For time dependent, static objects with a unit demand and one vehicle, the problem becomes a TDTSP. This problem can be used to model a real-life situation when there is only one helicopter that has to retrieve a set of static dispersed persons located in a certain region and the objective function is time-dependent.

When there is only one depot and a fleet of homogeneous vehicles with no limit on the tour length, the MMRO problem reduces to the GVRP. An example of a search and rescue operation that can be modelled as a GVRP corresponds to the rescue of a certain number of dispersed SAR objects (person in the water and liferafts with several persons aboard) who are drifting in the water and a fleet of helicopter is dispatched to retrieve them. In this example, if all the SAR objects correspond to individual persons in the water (cluster demand is one) them the MMRO problem becomes a GmTSP. If there is only one helicopter dispatched, the problem becomes a GTSP.

In order to have MMRO instances that approximate real-life scenarios it is relevant to consider a small time steps, usually steps with few minutes. This implies that the corresponding instances can be quite large in dimension which may be difficult to tackle with exact methods. The next Section describes an heuristic approach to the MMRO problem intended to produce better quality solutions for large instances where exact methods are unable to solve them.

2.3 Look-ahead methods for combinatorial optimization problems

For some combinatorial optimization problems that are $\mathcal{NP}\text{-}Hard$, it may be quite difficult to find the optimal solution due to the size of the problem or due to the time available to find a solution. In such cases, heuristic methods are a good alternative (sometimes, the only alternative) to obtain a feasible solution for our problem. The development of heuristic methods for large combinatorial optimization problems has been one of the most pursued topics by researchers worldwide in later and recent years and has been applied to several different types of problems. One of the most common and popular heuristic method is the construction heuristic, in which a solution to the problem is built step by step using a cost criteria until a complete and feasible solution to the problem is achieved at the end of the process. On a later survey on heuristic methods, Zanakis et al [182] analysed 442 papers and found that 155 were based on the use of construction heuristics.

Many construction heuristics are based on a greedy approach, in a sense that the "best choice" available is taken in each step of the process. This type of heuristic is very simple to develop and implement for several different types of problems. A classical example of a greedy construction heuristic is the Nearest Neighbour Heuristic (NNH) for the TSP.

Greedy heuristics usually do not guarantee optimal solutions (one exception is the greedy solution for the fractional knapsack problem) and in general they perform quite poorly.

One possible approach to overcome the crudeness of the greedy approach is to "look-ahead" and take into account how present choices will affect later choices of the heuristic. The idea of using a "look-ahead" strategy has been widely used by the artificial intelligence community, specially in the study of models for game-playing programs like chess and checkers (see Pearl [183]). Within the area of combinatorial optimization problems, earlier applications of the look-ahead strategy can be found in Atkinson [184], where he proposes a greedy look-ahead heuristic for a vehicle scheduling problem with time windows, and in Golenko-Ginzburg and Gonik [185] where a look-ahead procedure is used to solve the job-shop scheduling problem with random operations times. In Atkinson [184], the look-ahead feature is incorporated in the greedy heuristic via a greedy value function in which a costumer (who is being assessed at a certain stage of the algorithm) that is to be visited by a vehicle is measured not only by its immediate cost but also by the flexibility gained when taking into consideration other possible costumers that may be visited at later stages of the algorithm. The look-ahead technique has also been applied by Gemmil [186] to minimize the total makespan of resource-constrained projects and results showed that the look-ahead thecnique presented an average decrease of the duration of the projects between 5-8%. More recently, a similar strategy is used by Jin et al [187], where a look-ahead procedure is embedded within a greedy heuristic for solving a container relocation problem. Also, Akeb [188] presents a two-stage look-ahead based heuristic for the problem of packing spheres inside a three-dimensional bin of fixed dimensions and show that the results match or improve the majority of the best known solutions in the literature. These strategies are highly problem-tailored and not directly applicable to other problem variants or different types of problems.

The first look-ahead method that can be applied to any combinatorial optimization problem was first proposed by Duin and Voss [12] and is commonly known by the pilot method. The pilot method is presented as a 'tempered greedy algorithm based on look ahead results, pilots, obtained by heuristic repetition for each possible choice' [13, p. 286]. The idea is to use an heuristic, called pilot heuristic, within an heuristic approach to evaluate the merit of choices at each step where the merit is associated to a full grown solution that is conditional to that specific choice. The heuristic approach is not limited to construction heuristics, it can also be applied to procedures for improving solutions, like steepest descent or within a local search method. The term "pilot" is also used as an acronym meaning "Preferred Iterative LOok ahead Technique" or "Perform Improved Look ahead Objective-value Tests" [13, p. 286]. This method was first applied to the Steiner tree problem in graphs (see Duin and Voss [189]) which served as a vehicle of demonstration for the pilot method and later, the authors presented it as a metaheuristic suitable to solve any combinatorial optimization problem [12, p. 182]. Similar ideas to the pilot method were developed under different names, being the most famous the *rollout method* by Bertsekas et al. [190]. The article of Duin and Voss [13, p. 289] presents a survey on these similar methods.

The next subsection presents a formal description of the pilot method proposed by Duin and Voss [183, 185] for a generic combinatorial optimization problem, where the pilot procedure is a construction heuristic.

2.3.1 Pilot method for combinatorial optimization problems

One elementary condition to apply the pilot method to a combinatorial optimization problem consists in knowing an heuristic approach to that problem. One of the simplest approach is to use a construction heuristic. Taking the general case of a combinatorial optimization problem from Duin and Voss [12, pp. 182–183], consider a combinatorial optimization problem defined on a finite set of elements E weighted by a cost function $c: E \to \mathbb{R}$. The problem is to select at minimum cost a subset $S^* \subset E$, satisfying some feasibility properties. A heuristic $\mathbb{H}$ is available for the problem and is able to produce a feasible solution, where the objective value of the solution can deviate significantly from the optimal objective value. The idea of the pilot method is to build a partial solution (also designated by "master solution"), step by step, where heuristic $\mathbb{H}$ (acting as the "pilot heuristic" or subheuristic) is used to look ahead, in the sense that the element to be integrated in the partial solution (at each step) is the one with the most benefit of the fully grown solution obtained by the pilot heuristic. At each step of the pilot method, for every element $e \notin M$ (not in the partial solution) the pilot heuristic will extend a copy of M into a fully grown solution, $S(e)$, such that the element e is included. Let $c(e)$ denote the objective value of the solution $S(e)$ obtained by the pilot heuristic $\mathbb{H}$ for each $e \in E \backslash M$ and let e_0 be the most promissing element according to the heuristic $\mathbb{H}$, that is $c(e_0) \leq c(e)$ for all $e \in E \backslash M$. Element e_0 is included in the master solution according to some rule. The process may continue until the master solution is a feasible solution for the problem or further pilot calculations do not lead to further improvements. An interesting feature in the pilot method is the possibility to keep in memory the best fully grown solution found during the process. If computational times become undesirably large, one can allways stop the pilot method using an elapsed time criteria and return the best "pilot solution" found (assuming the master solution is not yet fully grown).

Algorithm 2.1 is one of the simplest illustrations of the pilot method for a combinatorial optmization problem. In each step of the pilot method an upperbound (assuming the problem is to minimize a cost funtion) on the optimal value can be calculated using the pilot heuristic. These values can also be used as a stop criteria if the pilot results do not improve when compared to the best pilot result in the previous step. In this situation, it is likely that the master solution is not a fully grown solution for our problem, but if the best pilot solution is kept in memory then the algorithm can retrive this solution as its output.

Algorithm 2.1 – Pilot method (constructive heuristic approach)

Inputs: partial solution M ("the master solution"), Tmax (maximum time available), pilot heuristic $\mathbb{H}$, set E, cost function c

Output: solution S^*

1: etime = 0 (elapsed time); $cmin = +\infty$ (minimum known cost)
2: While M is not fully grown and $etime \leq Tmax$
3:     For each $e \in E \backslash M$
4:         Use subheuristic $\mathbb{H}$ to obtain solution $S(e)$ with cost $c(e)$;
5:     Choose $e_0 = argmin\{c(e): e \in E\backslash M\}$; $M := M \cup \{e_0\}$; update $etime$;
6:     If $c(e_0) < cmin$
7:         $cmin = c(e_0)$;
8:         $S^* = S(e_0)$;
9: End while
10: If $c(M) < cmin$
11:     $cmin = c(M)$;
12:     $S^* = M$;

In a pure construction heuristic approach, the partial solution M, which is used as input to the pilot method, can be defined as the empty set. In such case, the first step in the pilot method would account to inspect all possible solutions that can be obtained by selecting each of the elements of the set E. In step 3 of algorithm 2.1, only one element $e \in E\backslash M$ is choosen to be incorporated into the master solution in order to use the subheuristic $\mathbb{H}$ to obtain solution $S(e)$ with cost $c(e)$. This particular step of the algorithm may incur in high running times for the overall algorithm since subheuristic $\mathbb{H}$ will be called as many times as the number of elements in the set $E\backslash M$ at a given step of the algorithm. This is, in fact, an undesirable feature of the pilot method and one of the challenges to make this method more successful. Several strategies have been proposed by Duin and Voss [13, Sec. 2.2.] to tackle the high running times of the pilot method. Some of these strategies provide different variants of the pilot method, which in turn provide different performances in accuracy and speed.

In the combinatorial optimization problem used by Duin and Voss it is not explicitly specified whether the order of the elements added to the master solution affects its cost or not. Problems such as the TSP or vehicle routing probems have the characteristic that the order by which the costumers are visited by the vehicles is relevant for the purpose of cost. For these problems, the pilot method can be changed to allow multiple elements to be added to the master solution with direct effect on the running time. If the elements are evaluated according to a sequence, one can set the number of elements to be selected on each position as well as the number of elements in the sequence (sequence length) to be added to the master solution. Adding a sequence of elements, with a certain length, to the master solution in each step of the pilot method amounts to perform a search in "depth" in a search tree where each branch is a sequence, or a partial sequence, of elements in $E \backslash M$. The number of elements to be selected in each position of the sequence does not require to be the same and can be defined as a "breadth" parameter for the number of elements inspected at a specific depth or level. The selection of the elements to be added to the master solution or to the sequence (that will be added to the master solution) is usually performed using a function or a measure. Taking the TSP as an example, the nearest neighbour heuristic uses a distance function to select a not yet included node into the solution. This implies that the function or measure to be used for selecting the elements to be added to the master solution can also affect the performance of the pilot method in terms of accuracy and speed. All these features limit the number of "fully grown" solutions to be evaluated in each iteration of the pilot method.

To illustrate these concepts, let us consider the set $E = \{1,2,3,4,5\}$ and the problem is to find a minimum cost sequence of all elements in E. This is a simple and small problem, since the number of possible sequences is $5! = 120$ and can be interpreted as a TSP. Let us suppose that the pilot method is used and, in each iteration, a sequence of two elements in E are added to the master solution. To limit the number of possible sequences, we consider a limit of two elements in the first component and three elements in the second component. The sequence length is the depth parameter, which in this case we have a two-level depth and a breadth vector of $(2,3)$. Figure 17 illustrates the sequences evaluated in the first iteration of the pilot method (in bold) and the discarded sequences (gray dashed lines). The sequences of elements to be added to the empty master solution are $\{(1,2), (1,4), (1,5), (3,1), (3,2), (3,4)\}$. These sequences will be extended to fully grown solutions using the pilot heuristic, which are designated by $S(1,2)$, $S(1,4)$, $S(1,5)$, $S(3,1)$, $S(3,2)$ and $S(3,4)$. The choice of the elements to be in the sequence is made by a function or measure. To choose the elements in each sequence it is necessary to evaluate all elements in each level with a given function or measure. This evaluation can also be very time consuming, depending on the function and the size of the set E.

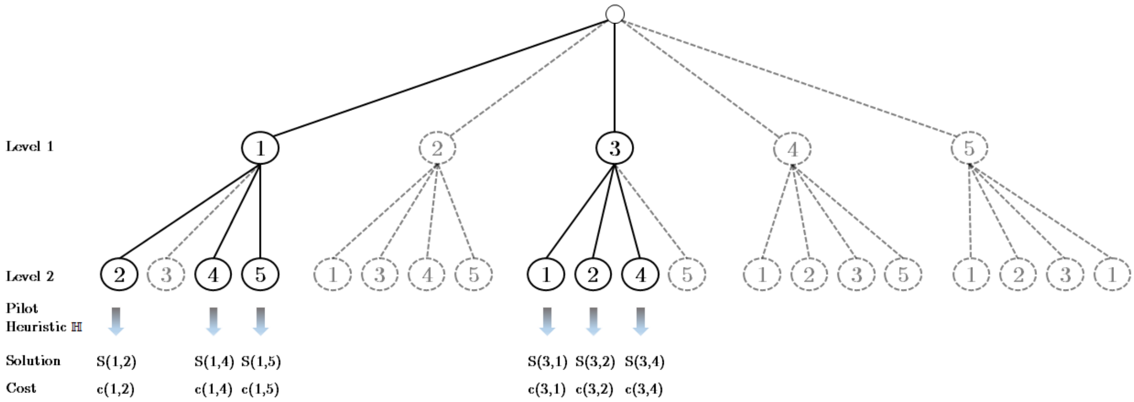

Figure 17. Search graph in iteration 1 of the pilot method for a two-level depth parameter with breadth vector $(2,3)$

The maximum depth of the search graph during the pilot method is limited by the number of elements in the set E. In each iteration the search graph will become smaller in terms of its nodes since the set M (master solution) will become larger. Therefore, it is expected that the number of fully grown solutions evaluated will decrease by each iteration.

After the evaluation of all of the fully grown solutions, one has to decide which element or set of elements, associated to the most promising fully grown solution, is going to be added to the master solution. If several elements or the sequence of elements itself is added, then it is expected that the pilot method will require few iterations to provide a final solution. Nevertheless, the running time of the pilot method depends greatly on the number of fully grown solutions evaluated in each step rather then the number of steps.

The pilot method can be interpreted as a guided search method, where in each iteration a guided search is performed in a graph that represents possible sequences of elements that can be added to a partial solution. The “guided” search is mostly determined by the function or measure used to select the elements of the set E for the sequence which will be added to the partial solution and extended to a fully grown solution by the pilot heuristic.

Once the most promising fully grown solution is identified, one has to decide which elements or elements are going to be added to the master solution to prepare the following iteration (see step 5 in algorithm 2.1). For sequence length d, several possibilities are available in this step: one can add the first element in the sequence, the first n elements of the sequence (with $n < d$) or the sequence itself ($n = d$). Choosing any element or sequence of elements different from the latter will not guarantee the cost associated with the respective fully grown solution. The simplest choice is to select the element in the first component of the preferred sequence. For example (see Figure 17), if the most promising fully grown solution is $S(3,2)$, then the element to be added to the master solution would be the element with number 3. Algorithm 2.2 describes the pilot method where a sequence of elements of length d and breadth vector b is evaluated and only the first element in the preferred sequence is added to the master solution.

Algorithm 2.2 – Pilot method (evaluation of sequences of elements)

Inputs: partial solution M ("the master solution"), Tmax (maximum time available), pilot heuristic $\mathbb{H}$, set E, cost function c; depth parameter d; breadth vector b

Output: solution S^*

etime = 0 (elapsed time); $cmin = +\infty$ (minimum known cost)
While M is not fully grown and $etime \leq Tmax$
    Build the set of sequences $\Gamma = \{\xi \in E^d : \xi(i) \in E' \subseteq E, |E'| \leq b(i)\}$
    For each sequence $\xi \in \Gamma$
        Use subheuristic $\mathbb{H}$ to obtain solution $S(\xi)$ with cost $c(\xi)$;
    Choose $\xi_0 = argmin\{c(\xi) : \xi \in \Gamma\}$; $M := M \cup \{\xi_0(1)\}$; update $etime$;
    If $c(\xi_0) < cmin$
        $cmin = c(\xi_0)$;
        $S^* = S(\xi_0)$;
End while
If $c(M) < cmin$
    $cmin = c(M)$;
    $S^* = M$;

In Algorithm 2.2, one of the key aspects stands with the procedure to build the set of sequences Γ. The size of set Γ depends greatly on the depth and breadth parameters. If the set E has n elements and we are interested in evaluating sequences with length d and a breadth parameter b (b is the vector with d components, $b = (b_1, b_2, \ldots, b_d)$), then the number of sequences to be evaluated in each iteration is limited by $\prod_{i=1}^{d} C_{b_i}^{n-|M|-i+1}$. In the example of Figure 17, where there is a two depth level and the breadth vector $(2,3)$, the total number of sequences evaluated in the first iteration of the pilot method is $C_2^5 . C_3^4$. These values show that the number of calls of the pilot heuristic is exponential, which stresses the importance of keeping this value under control in the pilot method.

The next subsection describes some of the available strategies to speed up the pilot method based on the design options that follows from the algorithm structure.

2.3.2 Design options within the pilot method

In each iteration of a pilot method a considerable amount of computational effort is required, leading to relatively high running times. Voss et al. [13, Sec. 2.2] discusses several strategies to speed up the pilot method. One of the proposed strategies is to resort to parallel processing for obtaining different pilot solutions simultaneously. In the same section, Voss et al. indicate other strategies to speed up the pilot method:

- Using a pilot heuristic with reduced time complexity.
- Limit the the number of iterations by modifying the master solution. This option amounts to add a sequence of elements to the master solution instead of only one element.
- Using a filtering approach to select a set of candidates of elements to be evaluated by the pilot heuristic instead of evaluating all of them.
- Restrict the pilot process to a certain *evaluation depth*.

The last alternative is refered in their paper and it aims to limit the master solution to a certain size and then completing it using a conventional greedy heuristic. Fink and Voss [191] applied this strategy to the flow-shop schedulling problem and results showed that the pilot method can be limited to reduced depths when running times are important.

Combining the previous alternatives with the nature of the combinatorial optimization problem will lead to different variants of the pilot method which in turn are expected to yield different performances depending on the preference between accuracy and speed. These alternatives can be generalized as design options to be applied to any combinatorial optimization problem. Taking the example of the combinatorial optimization problem used to describe Algorithm 2.2, several design options are available for obtaining a specific pilot method:

- Choosing the pilot heuristic $\mathbb{H}$.
- Instead of choosing only one element $e \in E \backslash M$ why not choose a sequence of elements $\xi = (e_1, e_2, \dots, e_d)$ to include in M, with $d \leq |E|$.
- If we are evaluating a sequence $\xi = (e_1, e_2, \dots, e_d)$ to include in M why not add a subsequence $\xi' = (e_1, e_2, \dots, e_u)$ with $u < d$ to the master solution M.
- Why evaluate all elements $e \in E \backslash M$ instead of evaluating a subset $A \subset E \backslash M$. This option is valid when only one element $e \in E \backslash M$ is added to the master solution.
- Using a diferent function or measure to choose the elements in two different situations:
    - When choosing the elements $e \in E \backslash M$ (see step 3 in Algorithm 2.1).

- When choosing the elements for building the sequences $\xi = (e_1, e_2, \dots, e_d)$ in the set Γ (see step 3 in Algorithm 2.2).

The first design option within the pilot method is the pilot heuristic. Using a different pilot heuristic leads to different variants of the pilot method. The pilot method can also be changed to use more then one pilot heuristic. For example, if there are no more gains in the evaluated pilot solutions, the method could resort to a different pilot heuristic to escape a possible local optimal solution. The second design option is a very interesting option for exploring the solution space of the combinatorial optimization problem. However, the number of possible sequences to evaluate is in general exponential in number. An interesting question to investigate is how to balance the sequence length and breadth in Algorithm 2.2 to achieve a better performance in accuracy when compared with Algorithm 2.1 (only one element is evaluated with the pilot heuristic). The third design option can shorten the number of iterations of the pilot method since the master solution is completed much faster. This option only guarantees a smaller number of iterations which in turn implies a smaller number of evaluated pilot solutions. The fourth design option is similar to the third strategy to speed up the pilot method proposed by Voss et al. [13, Sec. 2.2]. The final design option depends on the nature of the problem rather then the pilot method technicalities.

The combination of the presented design options allows a relatively large number of pilot methods variants for the MMRO problem which are described in subsection 3.6.2.

2.4 Summary

This Section highlights some of the most relevant facts and information regarding the topics related with the efficacy of the SAR System response to a maritime mass rescue incident and the mathematical models and algorithms used to cope with this problem:

i. There is an extensive material about search and rescue doctrine and new initiatives from independent organizations and also specialized organizations to identify new challenges and gaps in the SAR doctrine and ways to overcome them. One example is the MRO project initiative from IMRF.

ii. These initiatives have shown several gaps in the SAR capability when it is necessary to deal with extraordinary events that require MROs. Among these gaps are the need to identify areas remote from SAR facilities within the coastal state's SRR and also the need to assess the SAR capability to cope with MROs.

iii. There are large quantities of geospatial data regarding maritime traffic that can be used to identify areas remote from SAR facilities.

iv. The cruise ship industry is responsible for transporting millions of persons through maritime areas, including ocean areas on a daily basis through the entire globe. This poses an increase risk upon those lives since the number of passengers per ship is increasing. Coastal States should be prepared to deal with large scale incidents in maritime areas specialy in those areas that act as passage routes for large passenger ships.

v. A new threat is causing an increased apprehension among coastal states: terrorist attack on passenger ships. Recent events in Europe and South East Asia have placed the risk of a terrorist attack to targets such as cruise ships to an increased level. Such scenario will eventually lead to a MRO.

vi. States are recognizing MRO as a priority in their SAR response capability.

vii. The MMRO problem has similarities with several variants of the VRP problem, specially with GVRP, MDVRP and the VRP with profits.

viii. The concept of layered graph supports an integer linear formulation for the MMRO problem which provide two direct benefits:

   a. The movements of vehicles between time-indexed nodes satisfies the expected vehicles kinematics where the location of objects that are drifting in time is discritized and vehicles move between those locations at a constant speed. These constraints are implicit in the layered graph via the arcs feasibility.

   b. Since there can be no arcs between time-indexed nodes that have the same time index or prescribe a movement to the past, it follows that the ILP formulation does not require additional constraints to prevent unfeasible subcircuits in the solution tours.

ix. The pilot method is a relatively new technique for combinatorial optimization problems that, to the best of our knowledge, has not been thouroughly applied to vehicle routing problems. Nonetheless, the pilot method has a fairly simple mechanics and the potential to be straightforwardly applied to any combinatorial optimization problem when compared to other metaheuristics.

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Chapter 3

Maritime Mass Rescue Operations problem: Vehicle Flow Formulation and Heuristics

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3 Maritime Mass Rescue Operation Problem: Vehicle Flow Formulation and Heuristics

This Chapter presents the Maritime Mass Rescue Operation (MMRO) problem and describes the underlying parameters that are necessary to build an MMRO instance. The first Section describes the characteristics of the MMRO problem and how it is modelled as a routing problem in a graph. The second Section presents a vehicle flow model for the MMRO problem based on a huge layered graph where arcs between time-indexed nodes are feasible only if the ships or helicopters are allowed to travel within the given time ranges. The third Section presents several variants of two types of constructive heuristics where some of these variants can be considered to represent the priority rule followed by the rescuing vehicles when retrieving survivors from the water. The fourth Section discusses a pilot method to solve the MMRO problem that uses the constructive heuristic variants as pilot heuristics. The fifth Section describes a MATLAB prototype interface to construct MMRO instances and analyse the quality of the solutions that can be obtained by the different heuristics implemented in this dissertation. The sixth Section presents computational results for the different pilot method variants and constructive heuristics implemented. The last Section summarizes the major findings in this Chapter.

3.1 Modelling the MMRO problem

The purpose of modelling the MMRO problem and solving it is to provide means to assess the efficacy of the SAR system response to an incident through an MRO without having to deal with the real situation. Considering different scenarios for SRU availability and corresponding pre-location may give a thorough insight into how resources (availability and location) influence the success of the MRO. Different locations for the incident within ocean areas with higher density of passenger ships, may also provide a far-reaching awareness on the safety risk and expected effectiveness of the MMRO.

In a scenario where the available technology will provide the location through time of the survivors and their respective health conditions with high accuracy, then the rescue plan for the incident's MRO can be built from the MMRO solution. At the present moment the conceivable response to an incident that requires an MRO is based on the urgency to respond in order to minimize loss of life and it resumes to dispatch an adequate number of SRUs to the location of the incident and rescue all survivors as quickly as possible. If the system SRUs are too far away to provide assistance, the MRCC will identify nearby ships and call for their assistance. This is why nearby ships are essential to make part of the model. Especially in ocean areas where the presence of specialized SRU are very rare. In our model we want to estimate the survivor's trajectory through time, whether it is a single person in the water (PIW) or a liferaft with several persons inside. The term "survivor" is used throughout the text meaning a person that survived the incident and needs to be rescued (but it doesn't mean he will not die if not rescued in adequate time). A possible list of SAR objects and their respective drift parameters are given in figure N-3 in [2] and also in Annex A - List of SAR Objects and leeway values. Other objects must be considered in order to define a solution. Airbases for helicopters, the initial location of nearby ships, meeting locations for survivor's transfer and the replenishment's ship trajectory have to be considered. One way to model this is by considering a graph where nodes represent the location of objects with a certain time-stamp. These objects are grouped according to their characteristics.

3.1.1 A motivating example

To illustrate how we intend to model the MMRO problem let us consider a small example with the following characteristics:

- One nearshore naval base with one vessel and one helicopter that are dispatched at time t_0.
- The vehicles leave the depot at time t_0 and must end their tour in the depot.
- Three survivors are drifting and we know their locations at four time stamps which are 30 minutes apart.
- Each survivor location can only be visited by at most one vehicle. Once a survivor location is visited it implies the survivor is recovered.

The previous model, as described in Figure 18, encompasses a problem where two vehicles are dispatched to rescue three survivors who are drifting in the water. The nodes in the graph represent the survivor's location through time and the depot.

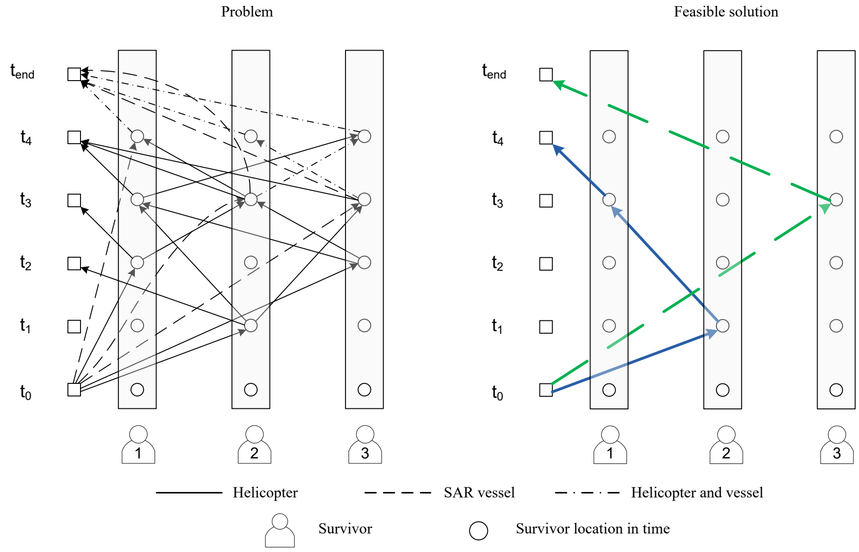

Figure 18. Example of small instance of the MMRO problem

The arcs represent direct transits between survivors and between survivors and the depot, for both helicopter and vessel. Both vehicles have different velocities and that can be checked by the time index upon which they arrive to each survivor. For example, the helicopter starts its tour at instant t_0 and if it goes directly to the location of survivor 2, it arrives there at time t_1, while the vessel can only arrive at the instant t_3. Since the survivors are in the water and subject to the maritime drift, it may happen that, with time, they approach each other or get further apart. These situations explain why the direct transit between the same survivors can have different time costs for the same vehicle.

The MMRO problem can be much more complex than the previous example. For instance, one must know in advance the location of each search object through time within a certain time step caused by the maritime drift. The following list resumes the characteristics of the MMRO problem that makes it a difficult problem to solve:

- Multiple vehicles can be considered with different speeds and capacities.
- Several depots for both helicopters and nearby vessel's acting as starting and ending nodes.
- Each object to recover may represent a single person in the water or a liferaft with dozens of individuals aboard.
- Hundreds of survivors and dozens of liferafts may be considered to have an approximate model of a real mass rescue incident.
- The benefit for rescuing a person is time-dependent.
- Replenishments may be considered in order to replenish helicopter and extend their range.

- The time required by a vehicle to recover a survivor must be taken into account in the problem.
- Drift calculations should be undertaken to obtain the location of each survivor or liferaft.
- Weather conditions should be taken into account when calculating the maritime drift and the performance of the vehicles for recovering survivors.
- Time step parameter should be taken as small as possible in order to reduce errors in travel calculations between nodes. This may cause the number of time-indexed nodes to be quite huge.
- It's not mandatory to recover all the survivors.

Since time is an important dimension in this problem we have chosen to model the MMRO problem as a routing problem over a layered graph where the nodes represent the location of objects in time and the arcs between time-indexed nodes are feasible only if the ships or helicopters are allowed to travel within the given time ranges. The MMRO problem can be interpreted as a Generalized Vehicle Routing Problem (GVRP) since each object is associated with a set of locations which only one of those should be visited by a vehicle.

The next two subsections describe how the nodes and arcs are calculated for an instance of the MMRO problem.

3.1.2 Node definition

The MMRO problem presents five categories of objects: helicopter depot, initial locations for nearby ships, meeting location for survivors transfer, replenishment ship trajectory and survivor's trajectory. Time is discretized in time-stamps which are equally set apart by a time step parameter. The time step can be set to seconds or minutes. The mission length is a parameter of the problem and, together with the time step, they outline the number of time-stamps $\boldsymbol{n}$.

The problem may have several objects of the same category. We may consider more than one airbase for helicopter departure or arrival. Several ships in the proximity of the incident may be committed to the rescue operations and a large number of survivors or rafts may be consider. Each node represents the location of an object through time.

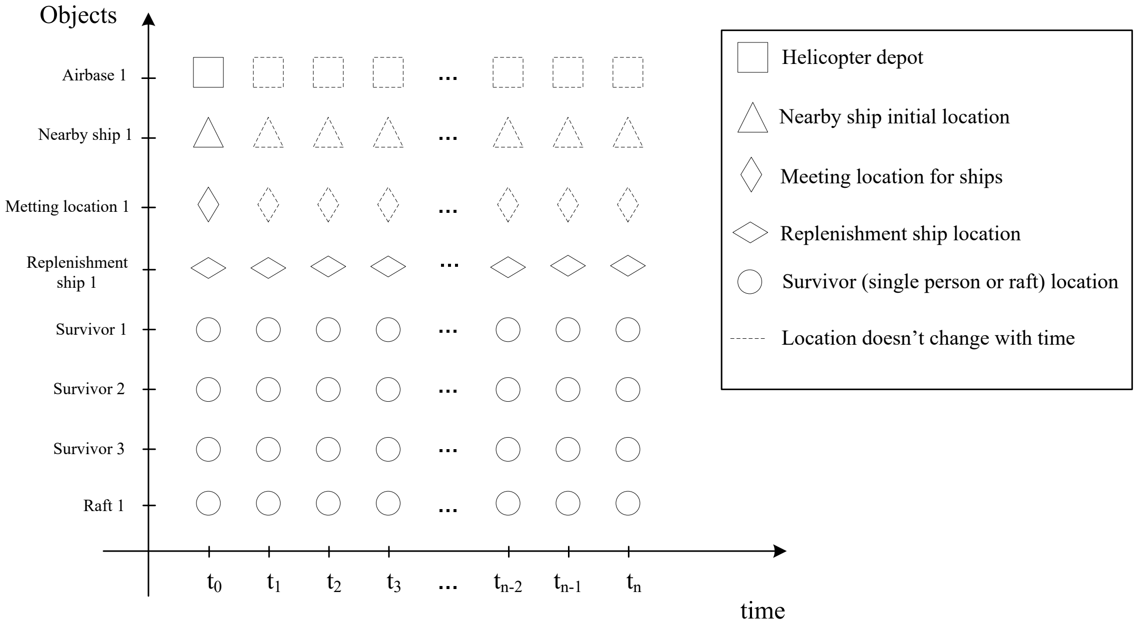

Figure 19. Node representation in layered graph for the MMRO problem

The total number of nodes depends on the number of objects and the number of time stamps, thus we have nodes indexed by time. Let F denote the set of airbases that serves as depots for helicopters. These depots serve as starting and finish locations for the helicopter's tour. Let N denote the set of initial location of nearby ships. Each location is assigned exclusively to one ship. Let M denote the set of meeting locations used to transfer survivors and are use as the finish depot for nearby ships. Let R denote the set of replenishment ships. Let S denote the set of survivors or liferafts. For n time stamps, the graph will have $|F| + |N| + |M| + n(|R| + |S|)$ nodes. Denoting by the letter P an instance of the MMRO problem, we can express all the objects of P by the sets $F(P)$, $N(P)$, $M(P)$, $R(P)$ and $S(P)$. When clear from the context, we may identify the sets omitting the letter P, for example, writing F instead of $F(P)$. The nodes in the layered graph represent the location of the objects through time. Only the objects from the sets R and S will change their location through time. This means the objects from sets F, N and M are depicted directly as nodes on the layered graph. The nodes representing the location through time of survivors or rafts in set S will be estimated by the objects drift.

Drift is the movement of an object caused by external forces. The most probable location of an object (usually referred as "search object"), corrected for movement over time, is known as the datum. Using wind forecasts that occur on a certain location and time one can calculate the forces acting on an object and estimate its drift. Maritime drift comprises four distinct forces: *Leeway* (LW), *Sea Current* (SC), *Wind Current* (WC) and *Tidal Current* (TC). Since the MMRO problem is set on ocean waters, a simplified drift model can be used to compute the object datum that only takes into equation the *leeway* and *wind currents*. Both of these forces can be estimated from wind forecasts for a certain location and time. To detail the survivors drift, we will make use of the following terminology and notation:

- Let t_0 be the initial instant of the incident. t_0 may also be considered as the time the MRCC receives the alert. Let τ_{t_0} be the geographic location specified in latitude and longitude degrees at time t_0. We have $\tau_{t_0} = (lat, lon)_{t_0}$.
- Let $LW_{\tau_{t_i}}$ be the local wind on datum τ_{t_i}. $LW_{\tau_{t_i}}$ describes the direction from where the wind blows (LW_{dir}) in degrees and its speed in knots LW_{spd} such that $LW_{\tau_{t_i}} = (LW_{dir}, LW_{spd})\ \tau_{t_i}$. The object's leeway for datum τ_{t_i} is $f\left(LW_{\tau_{t_i}}\right) = \left(f(LW_{dir}), f(LW_{spd})\right)_{\tau_{t_i}}$. The leeway drift speed $f(LW_{spd})$ can be consulted on Annex N in [2], figure N-2 and figure N-3, for several different types of search objects. The leeway drift direction $f(LW_{dir})$ represents the object's heading and is given by $f(LW_{dir}) = LW_{dir} + 180º$.
- Let $WC_{\tau_{t_i}}$ be the wind current on datum τ_{t_i}. $WC_{\tau_{t_i}}$ describes the direction the surface current takes (WC_{dir}) in degrees and its speed in knots WC_{spd} such that $WC_{\tau_{t_i}} = (WC_{dir}, WC_{spd})\ \tau_{t_i}$. To calculate $WC_{\tau_{t_i}}$ it is necessary to know the local winds on datum τ_{t_i}. Figure N-1 in Annex N [2] relates wind current direction and speed according to latitude and local wind speed.

Assuming T_M as the overall time the object is drifting (T_M corresponds to the mission period, or mission time window, which can also be stated as $T_M = t_n - t_0$), n as the number of iterations necessary to calculate the object's drift and α the time step ($\alpha = T_M/n$), the trajectory of the object, without variations on the leeway direction, can be described by the following equation:

$$\tau_{t_i} = \tau_{t_{i-1}} + \alpha\left[f\left(LW_{\tau_{t_{i-1}}}\right) + WC_{\tau_{t_i}}\right], i = 1, \dots, n \tag{3.1}$$

Equation (3.1) represents the vector sum between the object's location and the vector forces resulting from leeway and wind currents vectors at time t_i. τ_{t_n} represents the object's final position after being drifting. The sequence $\tau_{t_0}, \tau_{t_1}, \tau_{t_2}, \dots, \tau_{t_n}$ represents the object's trajectory from time t_0 until the last instant t_n.

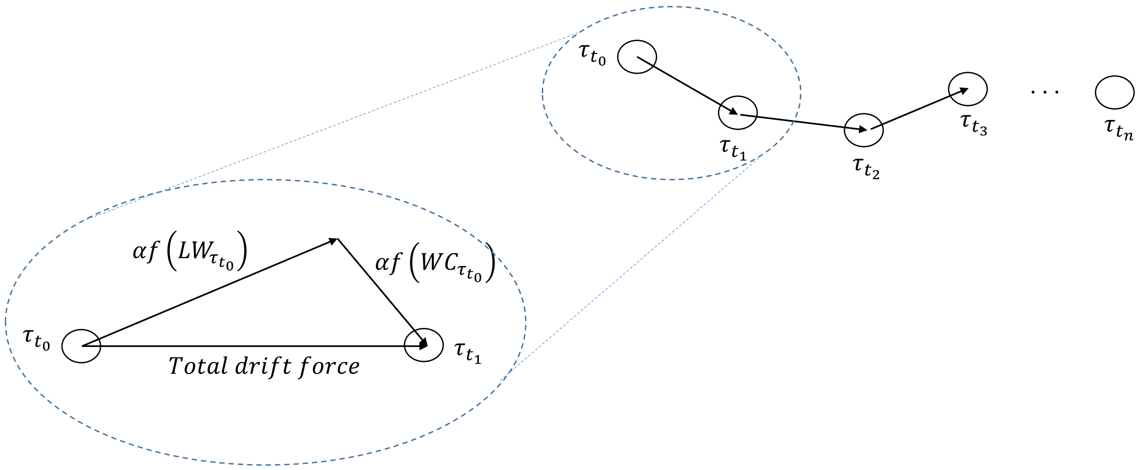

Figure 20. Vector plots for surface drift forces and datum sequence for drifting object

For building instances of the MMRO problem, a stochastic component associated with leeway will be considered for estimating the object's drift trajectory. In search planning, the stochastic component associated with leeway direction and speed is used (embedded in computer assisted search planning tools) to calculate a search area. Since our problem does not deal with the "search" problem, we use the stochastic parameters to estimate a single drift trajectory for each object in order to build the layered graph. Leeway direction varies according to the local wind and these variations have been estimated for several types of search objects (e.g. a raft with a shallow ballast and no drogue has an estimated leeway variation of $\pm 20º$ while a PIW with a scuba suit has a variation of $\pm 30º$). Without further information regarding the uncertainty of the leeway variation we assume a uniform distribution between the variation limits. The leeway intensity is estimated along with a probable error parameter for several search objects and local wind speed. The formula for the leeway intensity is resumed in the IAMSAR manual [2, Sec. 4.4] but more detailed information can be found in Allen et al. [192] and in Anderson et al. [193]. We also assume a uniform distribution associated with the leeway probable error. Let $f^*\left(LW_{\tau_{t_i}}\right)$ be the object's leeway vector with the random components for leeway variation and intensity, then,

$$f^*\left(LW_{\tau_{t_i}}\right) = \left(f(LW_{dir}) + U(-50,50), f\left(LW_{spd}\right) + \gamma\right)_{\tau_{t_i}} \tag{3.2}$$

The component γ represents the random variable associated with leeway intensity probable error and it is associated with the time step. The variations on the leeway intensity are modelled as:

$$\gamma = U[-0.35,0.35] \times f\left(LW_{spd}\right) \tag{3.3}$$

In equations (3.2) and (3.3) we present the values $\pm 50^{\circ}$ and ± 0.35 knots for leeway direction and leeway intensity variations respectively corresponding to a fishing vessel (length between 14-30 meters) of unknown type (see figure N-3, AIMSAR manual, Vol. 2). But these variations depend on the search object. The final equation for the drift trajectory is:

$$\tau_{t_i} = \tau_{t_{i-1}} + \alpha \left[f^* \left(LW_{\tau_{t_{i-1}}} \right) + WC_{\tau_{t_i}} \right], i = 1, \dots, n \tag{3.4}$$

An application was developed to set the initial location of each survivor or raft in the MMRO problem. The initial position at time t_0 is defined by the user. After the initial location of all objects regarding survivors are defined a routine is called to calculate their drift until the final instant t_n. Data regarding local winds can be obtained through GRIB files produced by the National Oceanic and Atmospheric Administration (NOAA) on a daily basis. GRIB (GRIdded Binary) is a mathematically concise data format used in meteorology to store historical and forecast weather data. It is a standardized data format by the World Meteorological Organization's Commission for Basic Systems, known under number GRIB FM 92-IX, described in WMO Manual on Codes No 206 (see [194]).

The replenishment ships trajectory are also defined by the user for each instant $t_i, i = 0,1,2, \dots, n$. Let T be the set of the time stamps defined by the user, then we have $T = \{t_0, t_1, \dots, t_n\}$, being t_0 the instant where the SAR system receives the alert and from which the maritime drift is calculated. The location of the replenishment ships serves only to increase the range of a helicopter that visit the respective node. During the planning stage the MRCC may dispatch replenishment ships to the incident's site in order to both help with rescue operations and also refuel helicopters. In our model the layered graph will contain time-indexed nodes associated with the replenishment ship's locations, which once visited by helicopters, will extend their range. Let S_k be the set of time-indexed nodes corresponding to the survivor's or raft datum k during its drift (total drift time is equal to the mission time window). Let R_k be the set of time-indexed nodes corresponding to the trajectory of the replenishment ship k during the mission time window. The set of all nodes in the layered graph for a problem P can be stated by the set $V = F(P) \cup N(P) \cup M(P) \cup R_1 \cup \dots \cup R_{|R(P)|} \cup S_1 \cup \dots \cup S_{|S(P)|}$. Let $\bar{R} = R_1 \cup \dots \cup R_{|R(P)|}$ and $\bar{S} = S_1 \cup \dots \cup S_{|S(P)|}$ be the sets of datum (which are also nodes) for all the replenishment ships and survivors. The nodes in the set V are sequenced according to the function f bellow:

$$f(i,t) = \begin{cases} i & i = 1, \ldots, |F| \\ |F| + i & i = 1, \ldots, |N| \\ |F| + |N| + i & i = 1, \ldots, |M| \\ |F| + |N| + (i-1)n + t & i = 1, \ldots, |R|, t = 1, \ldots n \\ |F| + |N| + |R|n + (i-1)n + t & i = 1, \ldots, |S|, t = 1, \ldots n \end{cases}$$

In a similar fashion, the objects are also sequenced according to the function g bellow:

$$g(i) = \begin{cases} i & i = 1, \ldots, |F| \\ |F| + i & i = 1, \ldots, |N| \\ |F| + |N| + i & i = 1, \ldots, |M| \\ |F| + |N| + |M| + i & i = 1, \ldots, |R| \\ |F| + |N| + |M| + |R| + i & i = 1, \ldots, |S| \end{cases}$$

The values of $f(i,t)$ may be used to check feasible locations to travel from (i,t) using the $f(i,t)$ row of the distance matrix and searching for positive values. Using f and g, every node $(i,t) \in V$ and object in the MMRO problem can be identified by a unique number which can be used to characterize a solution. This can be done using a double vector with the sequence of objects and node indexes.

3.1.3 Arc definition

The arcs between time-indexed nodes will only be feasible if the ships or helicopters are allowed to travel within the given time ranges. We assume that a vehicle visits only one location for a given object. This assumption states that a survivor is rescued only once or that the survivors in a raft are all rescued only once by the vehicle. This implies that there are no arcs between time-indexed nodes belonging to the same object. We also assume that vehicles cannot move to a location indexed to a time stamp earlier then the time stamp on the previous location (vehicles can't travel back in time). Both of these assumptions greatly simplify and reduce the number of arcs in our problem.

The distance in time between time-indexed nodes depends on the vehicle's speed. In the particular case of helicopters, the cruise speed is used to determine the distance in time between a pair of nodes corresponding to the location of different objects. The same rule applies for nearby ships. Helicopters won't visit the location of the initial position of nearby ships or the pre-determined meeting location. For a problem P, a helicopter can only move from nodes corresponding to the location of objects belonging to the sets $F(P)$, $R(P)$ and $S(P)$.

For a datum i ($i \in S_k$ or $i \in R_k$), $t^{\circ}(i)$ is the time stamp of datum i for the object k ($k \in S(P)$ or $k \in R(P)$). The time stamps associated with nodes represent departure instants for the vehicles. When a vehicle visits a node corresponding to a datum i associated to a survivor $k \in S(P)$, the time stamp $t^{\circ}(i)$ corresponds to the departure time associated with the datum i. This means that the vehicle at the time $t^{\circ}(i)$ has already rescued the survivor k and is ready to move to another location. If the vehicle moves to rescue a survivor $l \in S(P)$ after rescuing survivor k, then the time stamp $t^{\circ}(j)$ associated with the datum j should be close to the departure time from datum i, $t^{\circ}(i)$, plus the travel time between datum i and datum j and the time required to rescue survivor l. For helicopters, the time required to rescue a survivor includes the time the helicopter takes to correctly position itself over the survivor plus the time it takes to pull him out of the water. This time is also a parameter defined by the user and it usually depends on weather conditions. For good weather conditions and with a trained crew aboard the helicopter the recovery operation can take less than 10 minutes.

Since for every datum i, belonging to an object $k \in S(P)$, there can be only one arc to another set S_l, one needs to find the correct datum in S_l, for all $l \in S(P)$ and $l \neq k$. Moving from the location of an object k to the location of object l, the datum $j^* \in S_l$ should have a time stamp $t^{\circ}(j^*)$, such that:

$$t^{\circ}(j^*) = \underset{j \in S_l}{\operatorname{argmin}} \left\{ \left(t^{\circ}(j) - (t^{\circ}(i) + tr(i,j) + res\_time) \right)^2 \right\} \tag{3.5}$$

The problem in (3.5) should be solved for either helicopter and nearby ships and can be perceived as a kinematic calculation where the interception problem for two moving objects is solved. The term $tr(i,j)$ designates the travel time between datum i and datum j and it is obtained dividing the distance between i and j by the cruise speed of the helicopter or nearby ship. The term res_time designates the rescue time required by the vehicle to recover the survivor. The minimum in (3.5) is obtained when the difference between $t^{\circ}(j) - t^{\circ}(i)$ (time distance between i and j) and the travel time plus the recovery time is zero. Once more, this relation between datum will greatly reduce the number of feasible arcs between time-indexed nodes.

For helicopters, the arcs between nodes are defined for the following cases:

- Arcs from depots to nodes representing the location of a replenishment ship belong to the set

$$A_{F,R}^{h} = \left\{ (i,j) : i \in F(P), j \in R_k, k \in R(P), t^{\circ}(j) = \underset{j' \in R_l}{\operatorname{argmin}} \left\{ \left(t^{\circ}(j') - (t^{\circ}(i) + tr(i,j') + res\_time) \right)^2 \right\} \right\}$$

- Arcs from depots to nodes representing a datum of a survivor or raft belong to the set

$$A^h_{F,S} = \left\{(i,j) : i \in F(P), j \in S_k, k \in S(P), t^\circ(j) = \underset{j' \in S_l}{\operatorname{argmin}} \left\{\left(t^\circ(j') - (t^\circ(i) + tr(i,j') + res\_time)\right)^2\right\}\right\}$$

- Arcs from the location of replenishment ship to a node representing a depot belong to the set

$$A^h_{R,F} = \{(i,j) : i \in R_k, k \in R(P), j \in F(P)\}$$

- Arcs from the location of replenishment ship to a node representing a datum of a survivor or raft belong to the set:

$$A^h_{R,S} = \left\{(i,j) : i \in R_k, k \in R(P), j \in S_l, l \in S(P), t^\circ(i) < t^\circ(j), t^\circ(j) = \underset{j' \in S_l}{\operatorname{argmin}}\{(t^\circ(j') - (t^\circ(i) + tr(i,j') + res\_time))^2\}\right\}$$

- Arcs from the datum of a survivor or raft to a depot belong to the set

$$A^h_{S,F} = \{(i,j) : i \in S_k, k \in S(P), j \in F(P)\}$$

- Arcs from the datum of a survivor or raft to the location of replenishment ship belong to the set

$$A^h_{S,R} = \left\{(i,j) : i \in S_k, k \in S(P), j \in R_l, l \in R(P), t^\circ(i) < t^\circ(j), t^\circ(j) = \underset{j' \in R_l}{\operatorname{argmin}}\{(t^\circ(j') - (t^\circ(i) + tr(i,j') + res\_time))^2\}\right\}$$

- Arcs between survivor or raft datum's belong to the set

$$A^h_{S,S} = \left\{(i,j) : i \in S_k, k \in S(P), j \in S_l, l \in S(P), k \neq l, t^\circ(i) < t^\circ(j), t^\circ(j) = \underset{j' \in S_l}{\operatorname{argmin}}\{(t^\circ(j') - (t^\circ(i) + tr(i,j') + res\_time))^2\}\right\}$$

Denoting by A^h the set of arcs associated to helicopters, we have that $A^h = A^h_{F,R} \cup A^h_{F,S} \cup A^h_{R,F} \cup A^h_{R,S} \cup A^h_{S,F} \cup A^h_{S,R} \cup A^h_{S,S}$. For nearby ships, the arcs between nodes are defined for the following cases:

- Arcs from the nearby ship initial location to the datum of a survivor or raft belong to the set

$$A^{ns}_{N,S} = \left\{(i,j) : i \in N(P), j \in S_k, k \in S(P), t^\circ(j) = \underset{j' \in S_l}{\operatorname{argmin}} \left\{\left(t^\circ(j') - (t^\circ(i) + tr(i,j') + res\_time)\right)^2\right\}\right\}$$

- Arcs from the datum of a survivor or raft to a meeting location belong to the set

$$A^{ns}_{S,M} = \{(i,j) : i \in S_k, k \in S(P), j \in M(P)\}$$

- Arcs between survivor or raft datum's belong to the set

$$A^{ns}_{S,S} = \left\{(i,j) : i \in S_k, k \in S(P), j \in S_l, l \in S(P), k \neq l, t^\circ(i) < t^\circ(j), t^\circ(j) = \underset{j' \in S_l}{\operatorname{argmin}}\{(t^\circ(j') - (t^\circ(i) + tr(i,j') + res\_time))^2\}\right\}$$

Denoting by A^{ns} the set of arcs associated to nearby ships, we have $A^{ns} = A^{ns}_{N,S} \cup A^{ns}_{S,M} \cup A^{ns}_{S,S}$. Assuming we have all the nodes, corresponding to the locations in time of all objects, ordered by time stamp within their respective object set, we can define the adjacency matrix for helicopters and nearby ships, $\dot{A^h}$ and $\dot{A^{ns}}$, respectively:

$$\dot{A^h} = \begin{pmatrix} 0 & 0 & 0 & A^h_{F,R} & A^h_{F,S} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ A^h_{R,F} & 0 & 0 & 0 & A^h_{R,S} \\ A^h_{S,F} & 0 & 0 & A^h_{S,R} & A^h_{S,S} \end{pmatrix} \quad \dot{A^{ns}} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & A^{ns}_{N,S} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & A^{ns}_{S,M} & 0 & A^{ns}_{S,S} \end{pmatrix}$$

The number of columns and rows of $\dot{A^h}$ and $\dot{A^{ns}}$ correspond to the sequence of the nodes belonging to the objects of the sets F, N, M, R and S, respectively. Since the objects representing depots are not time-indexed then there must be an arc from every datum (survivor or replenishment ship) to a depot node to ensure the vehicle (helicopter or ship) can end its tour in a depot. The sets $A^{ns}_{S,M}$, $A^h_{R,F}$ and $A^h_{S,F}$ can be represented by all-ones matrix with adequate dimension to ensure that the vehicles end node is an adequate depot.

The adjacency matrix for helicopters and nearby ships is preceded by the calculus of their respective distance matrix. Calculating the maritime drift for all survivors is the first step to obtain the location of all the nodes in the layered graph. The trajectory of replenishment ships is defined by the user, so the location in each time stamp is known. Knowing the location of all the objects in each time stamp makes the calculation of the distance between each node possible. Since we have two types of vehicles, it's necessary to calculate the helicopter's distance matrix between nodes and the same distance matrix for nearby ships (assuming all helicopters have the same cruise speed and the same assumption for ships). Since helicopters and ships move with different speeds this will imply different travel times between datum. The distance matrix D will be calculated for all pairs of nodes relating all objects in the MMRO problem through the discretized mission time window. For a pair of nodes (i,j), the distance will only be calculated if $t^{\circ}(i) < t^{\circ}(j)$ (which causes the upper triangular sub-matrixes in the distance matrix). Since helicopter and nearby ships have different cruise speeds, the arcs between pairs of nodes are not the same. Their adjacency matrixes will be calculated using the distance matrix D in a two stage process. The first stage requires to calculate the travel time between a pair of datum belonging to different objects. The second stage comprises a kinematic calculation in order to find the correct datum j belonging to an object l that can be reached by a vehicle positioned at a datum i (where $t^{\circ}(i) < t^{\circ}(j)$) belonging to an object k. Let D^h and D^{ns} be the distance matrix for helicopters and nearby ships, respectively. The figure below shows the structure of the distance matrixes D, D^h and D^{ns}. The adjacency matrixes A^h and A^{ns} have a similar structure to the matrixes D^h and D^{ns}, respectively.

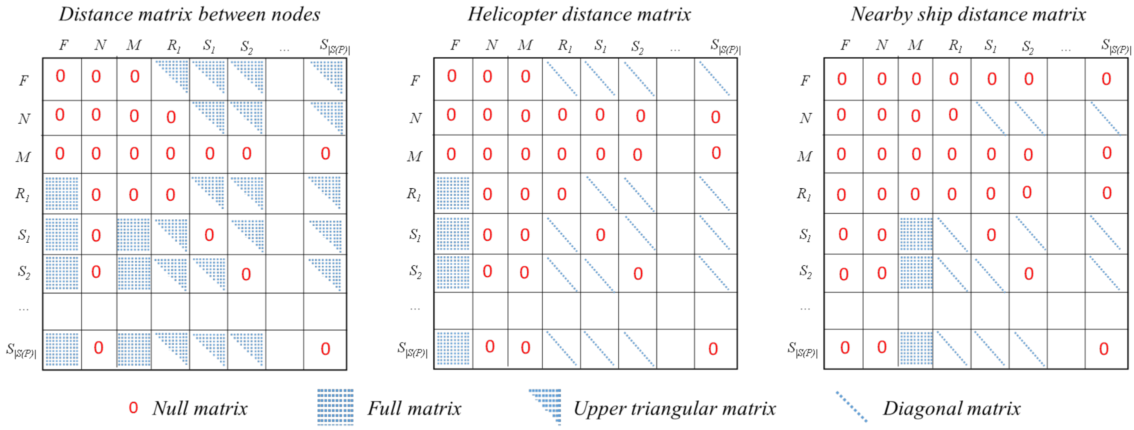

Figure 21. Distance matrixes $\mathrm{D}, \mathrm{D}^{\mathrm{h}}$and $\mathrm{D}^{\mathrm{ns}}$from left to right.

In the above matrixes, only one replenishment ship (R_1) was considered in order to simplify the presentation of the structure. The distance matrixes are relevant to build the constraints concerning operational range for helicopters. As for nearby ships it is possible to exclude arcs between datum where the travel time between them is greater than the mission time window.

The profit associated with rescuing a survivor is proportional to the time spent on water or on a liferaft. Predicting survival times for immersion victims is not a precise science [2, Para. 3.8.6] and there is no formula to determine exactly how long someone will survive. In our model we assume a maximum abstract value for rescuing a survivor that decreases with time. We also consider the possibility of retrieving a corpse which corresponds a smaller profit than that associated with rescuing a person alive. For each person we define two parameters that describe the profit of rescuing them. The first parameter defines the maximum profit for rescuing a person alive. The second parameter defines the profit for retrieving a deceased person. The profit variation through the mission time window requires that the survival time for each person involved in the incident has to be known. These times can be simulated based on historical data or simply randomly generated. The maximum profit will be associated with the initial instant t_0 and the profit of retrieving a dead person is associated with an instant $t_d^*(k)$ which is the time stamp where the person k dies. We assume that all survivors are alive at the instant t_0. The two profit parameters associated with rescuing a survivor are $prls$ (profit for rescuing a living survivor) and $prds$ (profit for rescuing a deceased survivor). The idea is that the profit is maximum at the earliest moment possible, that it at the instant t_0, and it decreases linearly to the value defined by $prds$ at the instant $t_d^*(k)$. After the instant $t_d^*(k)$ the profit remains constant.

3.1.4 Algorithm for building a MMRO problem instance

The next algorithm resumes the construction of an MMRO problem instance. The algorithm's major outputs are the data structures with the distance matrixes related to the type of vehicles available in the problem and the list of feasible arcs.

Algorithm 3.1 MMRO problem

Inputs: Number of Helicopters to employ, their characteristics (recovery time [*res_time*], range, passenger capacity) and their initial depot location, nearby ships initial location and characteristics (passenger capacity and recovery time), meeting locations, replenishment ships trajectory and refuelling time (*ref_time*), survivors initial position and characteristics (life expectancy through the mission time window), time step, mission time window, initial alert time stamp, weather forecasts for the location of objects along mission time window.

Output: Set of nodes V and its respective data table $\boldsymbol{T_V}$ (relating nodes with their location, time stamp, and type of object); distance matrixes $\boldsymbol{D^h}$ and $\boldsymbol{D^{ns}}$ for helicopters and nearby ships; Set of feasible arcs for all vehicles $\boldsymbol{A}$ and its respective data table $\boldsymbol{T_A}$ (relates arcs with nodes, cost, vehicle, and time stamps).

Calculate maritime drift for all objects representing survivors: return set V and $\boldsymbol{T_V}$;

Calculate distance matrix: return $\boldsymbol{D}$;

For each node $i \in F$

For each object $l \in R(P) \cup S(P)$

Find $j^* \in \bar{R} \cup \bar{S}$ that $t^\circ(j^*) = \underset{j' \in S_l}{\operatorname{argmin}}\{(t^\circ(j') - (t^\circ(i) + tr(i,j'; helicopter) + res\_time))^2\}$;

If $\sim isempty(j^*)$, do $D^h(i,j^*) = D(i,j^*)$;

For each node $j \in F \cup N \cup M$, do $D^h(i,j^*) = Inf$, $D^{ns}(i,j^*) = Inf$;

For each node $i \in N$

For each object $k \in S(P)$

Find j^* such that $t^\circ(j^*) = \underset{j' \in S_l}{\operatorname{argmin}}\left\{\left(t^\circ(j') - (t^\circ(i) + tr(i,j'; nearby\ ship) + res\_time)\right)^2\right\}$;

If $\sim isempty(j^*)$, do $D^{ns}(i,j^*) = D(i,j^*)$;

For each $j \in S_k$, $D^h(i,j) = Inf$;

For each node $j \in V \backslash \bar{S}$, do $D^{ns}(i,j^*) = Inf, D^h(i,j^*) = Inf$;

For each node $j \in M$, do $D^{ns}(i,j^*) = Inf$, $D^h(i,j^*) = Inf$;

For each node $i \in \bar{R}$

For each object $k \in S(P)$

Find $j^* \in S_k$ that $t^\circ(j^*) = \underset{j' \in S_l}{\operatorname{argmin}}\left\{\left(t^\circ(j') - (t^\circ(i) + tr(i,j'; helicopter) + ref\_time)\right)^2\right\}$;

If $\sim isempty(j^*)$, do $D^h(i,j^*) = D(i,j^*)$;

For each node $j \in V \backslash \bar{S}$, do $D^{ns}(i,j^*) = Inf$, $D^h(i,j^*) = Inf$;

For each node $i \in \bar{S}$

For each node $j \in F$, do $D^{ns}(i,j^*) = Inf$, $D^h(i,j^*) = D(i,j)$;

For each node $j \in N$, do $D^{ns}(i,j^*) = Inf$, $D^h(i,j^*) = Inf$;

For each node $j \in M$, do $D^{ns}(i,j^*) = D(i,j)$, $D^h(i,j^*) = Inf$;

For each object $k \in R(P)$

Find $j^* \in R_k$ that $t^{\circ}(j^*) = \underset{j' \in S_l}{\operatorname{argmin}} \left\{ \left(t^{\circ}(j') - (t^{\circ}(i) + tr(i,j'; helicopter) + res\_time)\right)^2 \right\}$;
If $\sim isempty(j^*)$, do $D^h(i,j^*) = D(i,j^*)$;
For each node $j \in V \backslash \{\overline{S} \cup \overline{R}\}$, do $D^{ns}(i,j^*) = Inf$, $D^h(i,j^*) = Inf$;
For each object $k \in S(P)$
Find $j^* \in S_k, j^* > i$, that $t^{\circ}(j^*) = \underset{j' \in S_l}{\operatorname{argmin}}\{(t^{\circ}(j') - (t^{\circ}(i) + tr(i,j'; helicopter) + res\_time))^2\}$;
If $\sim isempty(j^*)$, do $D^h(i,j^*) = D(i,j^*)$;
Find $j^* \in S_k, j^* > i$ that $t^{\circ}(j^*) = \underset{j' \in S_l}{\operatorname{argmin}}\{(t^{\circ}(j') - (t^{\circ}(i) + tr(i,j'; nearby\ ship) + res\_time))^2\}$;
If $\sim isempty(j^*)$, do $D^{ns}(i,j^*) = D(i,j^*)$;
Return matrixes $\boldsymbol{D^h}$ and $\boldsymbol{D^{ns}}$.
Calculate set $\boldsymbol{A}$ and data table $\boldsymbol{T_A}$ from all non-infinite entries in $\boldsymbol{D^h}$ and $\boldsymbol{D^{ns}}$.

The drift calculation is required only to obtain the survivors location through time. Equation (3.4) can be easily implemented to readily obtain the location of a drifting object given the right weather data. The calculus of the distance matrix is the most time consuming task in building an instance. To calculate the distance between two points in latitude and longitude a special function is required that should take into consideration the earth curvature.

The arcs in the set A cannot be used by every vehicle due to the feasibility constraint associated to travelling between time-indexed nodes. In order to check if an arc can be traversed by a vehicle one needs to check its distance matrix. Positive values in the distance matrixes represent feasible movements between time-indexed nodes (unfeasible arcs have a value of Inf[27] between its nodes).

The next figure illustrates the various objects of the MMRO problem with their respective nodes in the set V of the layered graph as well a feasible solution involving three vehicles:

[27] Inf represent the IEEE® value for infinity.

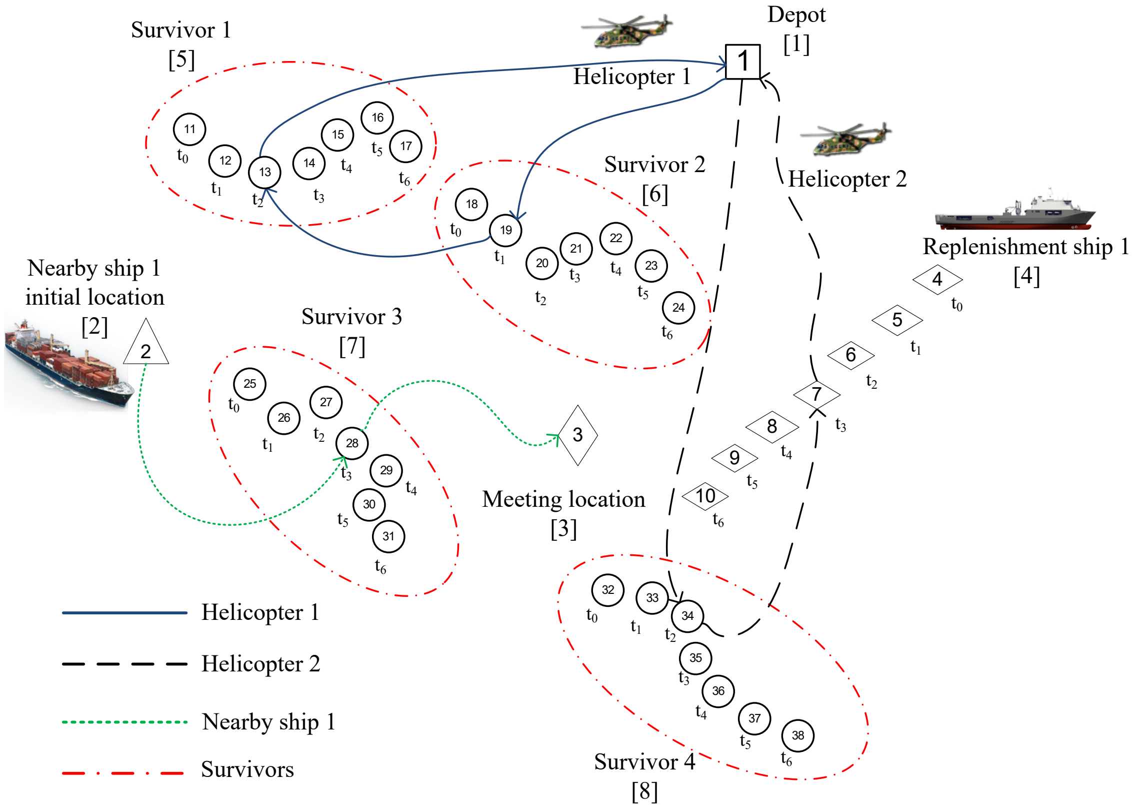

Figure 22. Diagram of objects and nodes in a MMRO problem

Let $\boldsymbol{vsk}$ be the line vector with the indexes of the objects for the MMRO problem and $\boldsymbol{vss}$ be the line vector with the indexes of the nodes in graph. In the above example the solution for vehicle 1 is represented by the line vectors $vsk\{1\} = [1\ 6\ 5\ 1]$ and $vss\{1\} = [1\ 19\ 13\ 1]$. Vehicle 2 is represented by $vsk\{2\} = [1\ 8\ 4\ 1]$ and $vss\{2\} = [1\ 34\ 7\ 1]$ and vehicle 3 is represented by $vsk\{3\} = [2\ 7\ 3]$ and $vss\{3\} = [2\ 28\ 3]$. Since the distance between every pair of nodes is known, it is possible to estimate the expected time arrival (ETA) of any vehicle to a depot and this feature makes it unnecessary to expand the depot nodes in time. The layered graph provides plentiful information regarding the survivors. The nodes in the vectors vss defines implicitly the time each survivor has spent in the water or the time it remained in the scene before being recovered. To calculate the vehicle's arrival instant to a depot it is only necessary to add the travel time between the depot and the last node to its respective time stamp. These simple calculations avoids the need to discretize in time the location of the depots (for both helicopter and nearby ships).

The MMRO problem presented so far is a routing problem that consists in finding k tours, one for each vehicle (both helicopter or nearby ship), in the direct graph $G = (V, A)$ that minimizes loss of life. Although the objective function relates loss of life with the time a survivor spends in the water, several statistical indicators can be considered to characterize the efficacy of the MRO from a particular solution. One can calculate the number of lives saved and relate it with the total number of lives to be rescued. We can also estimate the range covered by each vehicle and the time the survivors spent in the water until they were recovered. A desirable characteristic of the MMRO problem built with algorithm 3.1 rests in the fact that the graph $G = (V, A)$ is acyclic with regard to the nodes corresponding to survivors or replenishment ships.

In the next Section we propose a vehicle flow model for the MMRO problem based on a modified Picard and Queyranne formulation for the Time-Dependent Traveling Salesman Problem (TDTSP) [3]. We also present a constructive heuristic that mimic the "conventional" response of the SAR system and a pilot method to obtain better quality solutions for large scale instances.

3.2 Vehicle flow formulation

In the MMRO problem we consider k vehicles of two categories (helicopters and nearby ships) which have different capacities regarding the number of passengers aboard. For the helicopters, one may consider an average cruise speed when moving between nodes but it may well be necessary to consider different types of helicopters regarding their passenger capacity. In incidents where more than one SAR system dispatches aerial rescue units, we may have more than one type of helicopter involved in the rescue operations. With nearby ships we have the same predicament. The nearest ships may be a sailing vessel or a large cargo ship which can rescue several hundred of survivors (an example of such situation was the *Tampa affair*[28]).

Since vehicles have different characteristics we propose a vehicle based formulation built over the layered graph $G = (V, A)$ where the arcs are grouped according to the type of vehicle involved. The decision variables x_{ij}^{kpq}, which are binary, indicate whether vehicle k travels from node (i, p) to node (j, q), $x_{ij}^{kpq} = 1$ or not, $x_{ij}^{kpq} = 0$. The indices i and j refer to the problem objects and the indices p and q refer to time stamps. Since not all objects are discretized in time, we denote by T_i the set of time stamps of object i. For the sets of objects F, N and M we have $T_i = \{0\}, \forall\, i \in F \cup N \cup M$ and $T_i = \{0,1,2, \dots, n\}, \forall\, i \in S \cup R$. We consider nh available helicopters and ns available nearby ships. We shall use the variables x_{ij}^{kpq} for helicopters and the variables y_{ij}^{lpq} for nearby ships. The formulation is:

[28] On 24 August 2001, a 20 metre wooden fishing boat, the Palapa 1, with 438 (369 men, 26 women and 43 children) mainly Hazara unauthorised arrivals became stranded in international waters about 140 km north of Christmas Island. On 26 August, Rescue Coordination Centre (RCC) Australia, which had been aware of the vessel's distress requested all ships in the area to respond. Of the ships that acknowledged the request, the MV Tampa was closest to the site and began to proceed towards the distressed Palapa 1. All 438 survivors were recovered by the MV Tampa crew.

$$\text{(ILP1)} \quad \max z = \sum_{i \in V} \sum_{j \in V, j \neq i} \sum_{p=0}^{|T|} \sum_{q=p+1}^{|T|} \sum_{k=1}^{nh} c_{ij}^{pq} x_{ij}^{kpq} + \sum_{i \in V} \sum_{j \in V, j \neq i} \sum_{p=1}^{|T|} \sum_{q=p+1}^{|T|} \sum_{l=1}^{ns} c_{ij}^{pq} y_{ij}^{lpq} \tag{3.6}$$

$$\sum_{p=0}^{|T|} \sum_{i \in R \cup S} x_{1i}^{k0p} = 1 \qquad \forall\, k = 1, \dots, nh \tag{3.7}$$

$$\sum_{p=0}^{|T|} \sum_{i \in R \cup S} \sum_{j \in F} x_{ij}^{kp0} = 1 \qquad \forall\, k = 1, \dots, nh \tag{3.8}$$

$$\sum_{q=0}^{|T|} \sum_{j \in S} y_{lj}^{l0q} = 1 \qquad \forall\, l = 1, \dots, ns \tag{3.9}$$

$$\sum_{p=0}^{|T|} \sum_{i \in S} \sum_{j \in M} y_{ij}^{lp0} = 1 \qquad \forall\, l = 1, \dots, ns \tag{3.10}$$

$$\sum_{j \in S \cup F} \sum_{k=1}^{nh} \sum_{p=0}^{|T|} \sum_{q \in T_j} x_{ij}^{kpq} + \sum_{j \in S \cup M} \sum_{l=1}^{ns} \sum_{p=0}^{|T|} \sum_{q \in T_j} y_{ij}^{lpq} \leq 1 \qquad \forall\, i \in S \cup R \tag{3.11}$$

$$\sum_{j \in S \cup F} \sum_{k=1}^{nh} \sum_{p=0}^{|T|} \sum_{q \in T_j} x_{ji}^{kqp} + \sum_{j \in S \cup N} \sum_{l=1}^{ns} \sum_{p=0}^{|T|} \sum_{q \in T_j} y_{ji}^{lqp} \leq 1 \qquad \forall\, i \in S \cup R \tag{3.12}$$

$$\sum_{j \in V} \sum_{q \in T_j, q > p} x_{ij}^{kpq} = \sum_{j \in V} \sum_{q \in T_j, q < p} x_{ji}^{kqp} \qquad \forall\, i \in S \cup R, \forall\, p \in T_i, k = 1, \dots, nh \tag{3.13}$$

$$\sum_{j \in V} \sum_{q \in T_j, q > p} y_{ij}^{lpq} = \sum_{j \in V} \sum_{q \in T_j, q < p} y_{ji}^{lqp} \qquad \forall\, i \in S, \forall\, p \in T_i, l = 1, \dots, ns \tag{3.14}$$

$$\sum_{i \in V} \sum_{j \in V, j \neq i} \sum_{p \in T_i} \sum_{q \in T_j, q > p} d_{ij}^{kpq} x_{ij}^{kpq} - \sum_{i \in R} \sum_{j \in V, j \neq i} \sum_{p \in T_i} \sum_{q \in T_j, q > p} r_{ij}^{pq} x_{ij}^{kpq} \leq Aut_k \qquad k = 1, \dots, nh \tag{3.15}$$

$$\sum_{i \in V} \sum_{j \in V, j \neq i} \sum_{p \in T_i} \sum_{q \in T_j, q > p} u_i x_{ij}^{kpq} \leq Cap_k \qquad k = 1, \dots, nh \tag{3.16}$$

$$\sum_{i \in V} \sum_{j \in V, j \neq i} \sum_{p \in T_i} \sum_{q \in T_j, q > p} u_i y_{ij}^{lpq} \leq Cap_l \qquad l = 1, \dots, ns \tag{3.17}$$

$$x_{ij}^{kpq} \in \{0,1\} \qquad \forall\, i, j \in V, i \neq j, \forall\, p \in T_i, \forall\, q \in T_j, k = 1, \dots., nh \tag{3.18}$$

$$y_{ij}^{lpq} \in \{0,1\} \qquad \forall\, i, j \in V, i \neq j, \forall\, p \in T_i, \forall\, q \in T_j, l = 1, \dots., ns \tag{3.19}$$

Constraints (3.7) and (3.8) ensure that all helicopters leave and return to a depot. In constraint (3.7) it is assumed that all helicopters depart from depot "1" at time "0". Constraints (3.9) and (3.10) ensure that all nearby ships start their tour on their initial location at time "0" and finish their route at a pre-determined meeting location. Constraints (3.11) and (3.12) guarantees that each survivor or replenishment ship is not visited more than once by a single vehicle. Constraints (3.13) and (3.14) guarantees flow conservation for each node in the graph G. Constraint (3.15) states the maximum range for helicopters. Constraints (3.16) and (3.17) impose limits on the number of passengers each vehicle can have aboard. The parameter $u_i, i \in S$, represents the number of persons related to object $i \in S$. If the object is a single person in the water, then $u_i = 1$, but if the object is a liferaft or lifeboat, then $u_i \geq 1$. For these equipments, the parameter u_i is limited by their maximum capacity.

The above formulation does not require constraints to avoid sub circuits among vehicles due to the fact that the graph $G = (V, A)$ (upon which the variables are defined) is acyclic regarding nodes corresponding to survivors or replenishments ships[29].

Note that the formulation is a generalization of the Picard and Queyranne [169] formulation for the TDTSP if we consider only one vehicle, only one depot, no replenishments ships, only the time dependent constraint ($q = p + 1$, no kinematic constraint between datums) and a total number of time stamps equal to the number of survivors to be rescued.

If we consider only one time stamp ($|T| = 1$, the indices p and q would be suppressed from the formulation) the problem becomes a capacitated vehicle routing problem (CVRP). Additionally, if one considers only one vehicle we have the capacitated traveling salesman problem (CTSP).

The size of the problem depends greatly on the number of time stamps considered, the number of survivors and the number of vehicles. In a mass rescue operations it is expected a large number of survivors to be recovered and a lesser number of available vehicles to recover them. In ocean areas, the number of nearby ships can be quite scarce compared to a near shore incident. The next table presents different problem sizes depending on the number of vehicles, time stamps and survivors.

Table 1. Size of inputs for different instances of the MMRO problem.

time stamps (1)	helicopters (2)	helicopter depot (3)	nearby ships (4)	meeting locations (5)	replenishment ships (6)	number of survivors (7)	nº of nodes (8)	nº of arcs for each helicopter (9)	nº of arcs for each ship (10)	total number of arcs (11)
n	k	F	N	M	R	S	F+N+M+n*R+n*S	FR+FS+nRF+(n-1)RS +nSF+(n-1)SR+(n-1)S(S-1)	NnS+nSM+(n-1)S(S-1)	(9k+10N)
36	2	2	1	1	0	4	148	716	568	2000
72	2	2	1	1	0	4	292	1436	1144	4016
72	2	2	1	1	1	4	364	2150	1144	5444
144	2	2	3	1	0	100	14406	1444700	1459200	7267000
144	2	2	3	1	0	300	43206	12914100	12957600	64701000

In the above table, the number of arcs associated to each helicopter correspond to the arcs from the set of depots to the sets of replenishment ships and survivors ($|F||R| +$

[29] One can have a solution with sub circuits if the starting and end depot are the same. Sub circuits may occur in a solution only for helicopters that start and finish their route in the same depot. Sub circuits involving only survivors or replenishments ships are not possible due to the violation of the time dependent constraint which implicitly resides in the adjacency matrix for each vehicle in the graph G.

$|F||S|$), arcs from the replenishments ships to survivors and depots ($n|R||F| + (n-1)|R||S|$), arcs from the set of survivors to the replenishments ships and depots ($n|S||R| + (n-1)|S||F|$) and, finally, arcs between the sets of survivors ($(n-1)|S|(|S|-1)$). The component $(n-1)|S|$ in the previous term corresponds to suppressed arcs that start from the last position of a survivor (position on time stamp t_n), due to the constraint that arcs can only exist between the location of datum (i,p) and (j,q) if $q > p$.

An MMRO problem with a hundred SAR objects will easily have over 7 million variables. We would like to solve up to one hundred SAR objects and compare the results of applying heuristics that mimic the "conventional" response with near optimal solutions.

3.3 Constructive heuristics

In this Section two constructive heuristics schemes are presented for the MMRO problem. Both heuristics follow an improvement approach where a partial solution is built by including a feasible vehicle/survivor assignment in each iteration. The vehicle's routes are built simultaneously in both heuristics. The term vehicle/survivor assignment will be used throughout the dissertation to represent an assignment of a particular vehicle to a particular SAR object, which may represent a liferaft with several persons inside. When clear of context, the term "survivor" will be used when referring to an assignment between a SAR object and a rescue vehicle. When the context demands, the SAR object will be specified according to the list of SAR objects described in Annex A - List of SAR Objects and leeway values. The possibility of replenishment is not addressed in the constructive heuristics. Although the MMRO problem considers replenishment ships and their respective trajectories, these were not implemented in the heuristics due to the complexity in modelling the helicopter power-margin in these operations, which is discussed in Chapter 5. An example with replenishment ships is given in Chapter 4 and the challenges for their implementation in the constructive heuristics is discussed in Chapter 5.

The second heuristic distinguishes from the first by being greedier in the process of choosing the vehicle/survivor assignment. Several different criteria can be used on both heuristics for choosing a survivor to be rescued by a certain vehicle. For instance, if survivability information is available, it can be used to choose an assignment that specifies which survivor should be rescued by a certain vehicle. Distance in space or distance in time between a vehicle and a survivor can also be used as a decision criteria.

Since we consider autonomy constraints for rotary-wing vehicles (helicopters) and a time period to assess the efficacy of the SAR response to an incident, it may not be possible to recover all the survivors within the given time and all vehicles must end their tour on an adequate depot. This feature of the MMRO problem may condition the feasibility for a given vehicle to recover a certain survivor. In the MMRO problem, a **feasible vehicle/survivor assignment** between a vehicle k and a survivor s verifies the following conditions:

- Vehicle k can travel between its current position to the location of the survivor s within the mission time window.
- Vehicle k can return to a depot after recovering survivor s (doesn't violate the autonomy constraint).
- Survivor's weight does not exceeds the remaining capacity of vehicle k.

The first heuristic assumes that all the vehicles are ordered and tries to rescue survivors by evaluating sequentially each vehicle and, if possible, assign it to a survivor, according to one criteria. After an assignment is made, the next vehicle is analysed for a possible assignment. The method continues until all survivors are rescued or all vehicles have exceeded their capacity. In each step, the heuristic evaluates $ns - k$ possible assignments, with ns being the total number of survivors and k the number of previous steps. The solutions obtained by this method tend to equally distribute survivors through available vehicles.

The second heuristic evaluates, in each iteration, all possible feasible assignments between all available vehicles and survivors, and chooses the assignment with the highest score within the decision criteria used. This scheme is much more time consuming that the one in the first heuristic since in each iteration all feasible vehicle/survivor assignments has to be evaluated. With the merit of all feasible assignments all that remains is to choose the survivor with the highest priority (and its respective vehicle) using available information (if survivability times are considered, we can choose the survivor with the smaller amount of time to live that can still be rescued alive).

Figure 23 shows the procedure to check the feasibility of the vehicle/survivor assignment:

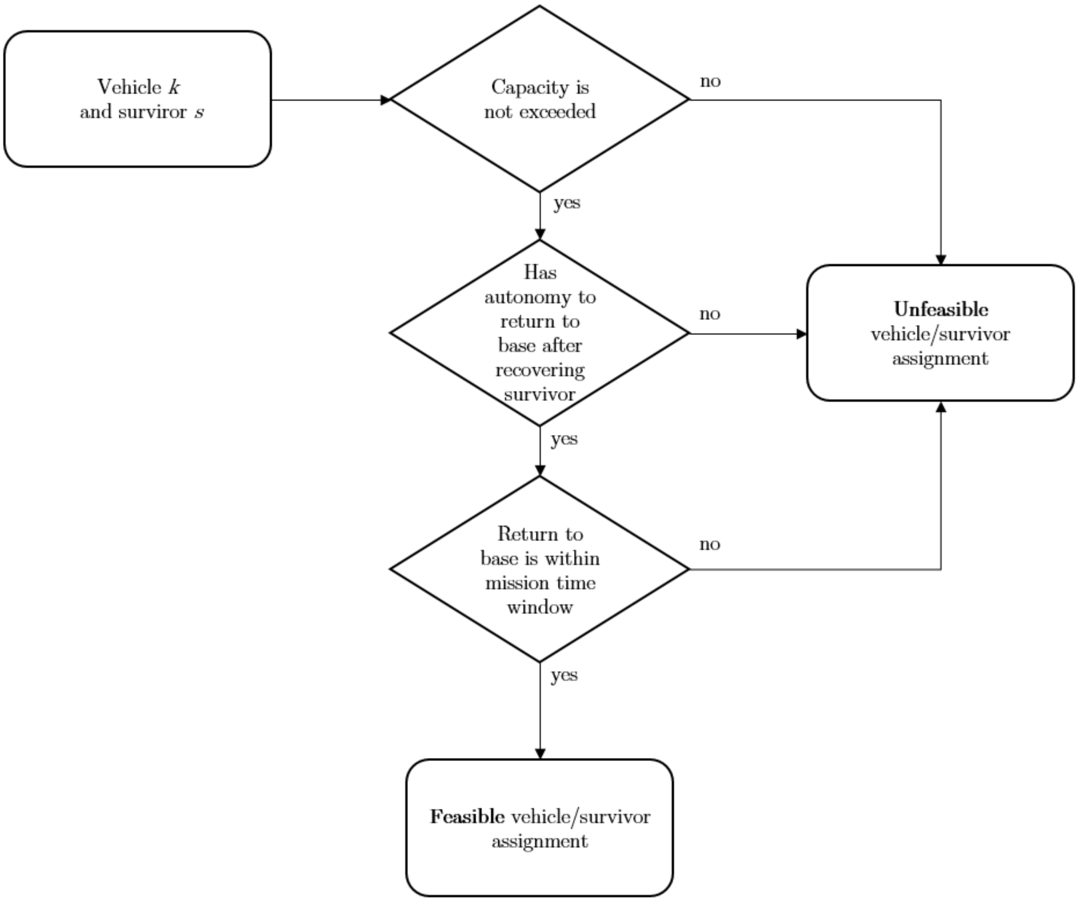

Figure 23. Procedure to check vehicle/survivor assignment feasibility

3.3.1 Criteria based on merit function for choosing a feasible vehicle/survivor assignment

In a real rescue operation the task of co-ordinating several rescue units is made by the SMC. The SMC must make some difficult decisions based on the best information available to provide guidance for the SRU on how they should procede in rescuing survivors who are drifting in the water. The most common scenario in a MRO is one were there is lack of information regarding the incident's victims location and seriousness of the survivor's heatlh condition. In the MMRO problem it is assumed that the location of survivors is known to all rescue units and also to the SAR system. If we also add the assumption that the survival times are also available, then several criteria can be used to assess or measure and compare different possible vehicle/survivor assignments:

- Profit gained by rescuing a survivor (directly related to the remaining lifetime (RLT) expectancy of the survivor upon the vehicle's arrival);
- distance in space between vehicle and survivor;

- expected time arrival (ETA) between the current vehicle's location and the location of the survivor upon the vehicle's arrival;
- cost of the solution obtained by completing the current partial solution using another heuristic.

Different criteria will produce different heuristics and consequently possible different solutions for the same problem. For a vehicle k located at the node (i, p) and a survivor s located at node (s, q) (we assume the vehicle can move from (i, p) to (s, q) according to its distance matrix), we shall denote by $\varphi(k, (s, q))$ the merit of the vehicle/survivor assignment for a given criteria. The merit of the assignment accounts implicitly for its feasibility. We shall use $\varphi(k, s)$ instead of $\varphi(k, (s, q))$ to simplify notation whenever it is not required to explicit the time component q associated with the object s. A vehicle/survivor assignment is feasible only if the vehicle can return to a depot after rescuing the survivor. Otherwise, the assignment is not feasible. For a vehicle k we have $\boldsymbol{rc}(k) = 0$ if there are no more survivors to rescue or if for all remaining survivors it is not possible to return to a depot after their rescue. If several depots are available, the feasibility of the assignment can be checked using the nearest depot after visiting the survivor location. The graph $G = (V, A)$ and the distance matrices for different types of vehicles are the main inputs for both heuristics.

3.3.2 Simple sequential constructive heuristic

The first constructive heuristic, for short **HC1**, builds a solution by assigning a survivor to a vehicle and repeating the process following the serial order of the vehicles. If we have $\boldsymbol{k}$ vehicles and $\boldsymbol{n}$ survivors $(n > k)$, in the first iteration there will be an assignment to vehicle 1, in iteration 2 there will be an assignment to vehicle 2 and the process repeats itself after vehicle k is assigned to a survivor until there are no more survivors to recover or all vehicles have exceeded their capacity or range. The procedure of assigning SAR objects to vehicles is very close to the "standard" procedure that the search and rescue mission co-ordinator (SMC) or the on-scene co-ordinator (OSC) would implement at the incident area and these are related to the "urgency of response" [2, Para. 3.8.5]. The urgency of response states that for a known distress, a SAR facility, preferably the craft closest to the scene or the quickest response SRU, should be immediately dispatched to confirm the distress position and recover persons in distress.

Algorithm 3.2 – HC1 Constructive Heuristic 1 for the MMRO problem

Inputs: Number of vehicles to employ $\boldsymbol{U}$, their characteristics (range, passenger capacity $\boldsymbol{Q_k}$); set of SAR objects $\boldsymbol{S}$ and weight $d_i, i \in S$; distance matrixes $\boldsymbol{D^h}$ and $\boldsymbol{D^{ns}}$ for helicopters and nearby ships; Merit function $\boldsymbol{\varphi}$.

Output: solution vectors $\boldsymbol{vss}$ and $\boldsymbol{vsk}$ for each vehicle, solution cost

Initialize vectors $\boldsymbol{vss}$ and $\boldsymbol{vsk}$ with the initial starting node for each vehicle;
While $|S| > 0$ *and* $\sum_{k=1}^{U} \boldsymbol{rc}(k) > 0$
  For $k = 1{:}U$
    $(i^*, p^*) = \max_{i \in S, p \in S_i} \{\varphi(k, (i,p)) : D^k(vss\{k\}(end), f(i,p)) > 0, Q_k > d_i\}$;
    If $i^* \neq \emptyset$
      $vss\{k\} = [vss\{k\}(end)\ p^*]$; $vsk\{k\} = [vsk\{k\}\ i^*]$;
      $S \leftarrow S \backslash \{s\}$; $Q_k = Q_k - d_{i^*}$
    Else $\boldsymbol{rc}(k) = 0$; End if
  For $k = 1{:}U$
    Examine vehicle k and terminate route if $\boldsymbol{rc}(k) = 0$ or $|S| = 0$
End while
For $k = 1{:}U$
  Examine vehicle k and terminate route if $\boldsymbol{rc}(k) = 0$ or $|S| = 0$
Calculate cost of solution and return $\boldsymbol{vss}$ and $\boldsymbol{vsk}$

This heuristic uses two logical conditions for the process to end and provide a feasible solution. The first logical condition verifies if there are no more survivors left to rescue. This can be easily verified if the set of survivors S is empty ($|S| = 0$). But it may also happen that the dispatched vehicles to the scene are not able to recover all the survivors within the mission period. This may happen because their capacity has been exceeded or their operational range does allow them to rescue a survivor and return to a depot with safety. In the particular case of helicopters it is paramount that they don't run out of fuel during rescue operations. The vehicles *capability to rescue* a survivor and still being capable to return to a depot afterwards is represented by a binary line vector $\boldsymbol{rc}$ with as much columns as the total number of vehicles. For a vehicle k, $\boldsymbol{rc}(k) = 1$ means that there is at least one survivor that can be rescued by vehicle k and afterwards it can end its route at a depot. When $\boldsymbol{rc}(k) = 0$ it means that vehicle k cannot be assigned to rescue remaining survivors. Naturally, the heuristics stops if $\sum_{k=1}^{U} \boldsymbol{rc}(k) = 0$ (all vehicles cannot rescue any more survivors).

3.3.3 Greedy sequential constructive heuristic

The second constructive heuristic, for short **HC2,** builds a solution in a two-step way. First it assigns one survivor to each available vehicle in order to satisfy the problem constraint where each vehicle must rescue at least one survivor. In the second step, the merit of all feasible vehicle/survivor assignments and also a function that states the priority associated to each remaining survivor are calculated. The chosen vehicle/survivor assignment to incorporate the solution corresponds to the highest merit vehicle/survivor assignment of the most priority survivor that hasn't been rescued. While **HC1** only requires the merit of the vehicle/survivor assignment for a specific vehicle at each step, **HC2** uses the survivor priority concept. The concept of priority is usually associated with health conditions of the survivors whenever this information is available for the vehicles and the MRCC. For example, the priority can be stated by the remaining survival time of each survivor among those who still can be saved alive. At each time step it is possible to have some information of the survivor that are still alive and those who have deceased. Normally a deceased survivor floats in the water horizontally while a living one remains vertically. If a vehicle can view a deceased survivor and a living one, the choice of who is the first to recover is without question the living one. This is why is makes sense to use the information regarding whether a survivor is still alive or deceased. This is a 0/1 information quite different from the expected remaining time alive. When this information is not available the priority can be assessed by the distance or ETA of the survivor to the nearest vehicle.

Algorithm 3.3 – **HC2** Constructive Heuristic 2 for the MMRO problem

Inputs: Number of vehicles to employ $\boldsymbol{U} = \boldsymbol{nh} + \boldsymbol{ns}$, their characteristics (range, passenger capacity $\boldsymbol{Q_k}$); set of SAR objects $\boldsymbol{S}$ and weight $d_i, i \in S$; distance matrixes $\boldsymbol{D^h}$ and $\boldsymbol{D^{ns}}$ for helicopters and nearby ships; Merit function $\boldsymbol{\varphi}$.

Output: solution vectors $\boldsymbol{vss}$ and $\boldsymbol{vsk}$ for each vehicle, solution cost

```
Initialize vectors vss and vsk with the initial starting node for each vehicle;
For k = 1:U
    (i*, p*) = max_{i∈S, p∈S_i} {φ(k, (i, p)): D^k(vss{k}(end), f(i, p)) > 0, Q_k > d_i};
    If i* ≠ ∅
        vss{k} = [vss{k}(end) p*]; vsk{k} = [vsk{k} i*]; S ← S\{i}
    Else rc(k) = 0; End if
While |S| > 0 and ∑_{k=1}^{U} rc(k) > 0
    For k = 1:U
        For all s ∈ S
            Calculate φ(k, s); If φ(k, s) = ∅ ∀ s ∈ S, then rc(k) = 0; End if
    For all s ∈ S
        Determine the priority of survivor s, ω(s)
        s* = max_{s∈S}{ω(s)}; (k*, p*) = max_{k=1,…,U, p∈S_{s*}} {φ(k, s*): D^k(vss{k}(end), f(s*, p)) > 0, Q_k > d_{s*}}
        If k* ≠ ∅
            vss{k*} = [vss{k*}(end) p*]; vsk{k*} = [vsk{k*} s*]; S ← S\{s}; Q_{k*} = Q_{k*} − d_s
        Else rc(k) = 0; End if
End while
For k = 1:U
    Examine vehicle k and terminate route if rc(k) = 0 or |S| = 0
Calculate cost of solution and return vss and vsk
```

These heuristics have in common the fact that in each iteration, a partial solution is built until a feasible solution is obtained. The difference between them rests in the number of vehicle/survivor assignments that are evaluated and the process for choosing the one to include in the partial solution. Both heuristics will be well defined only after we specify the merit and priority functions. Note that we have considered the maximum of the merit function when choosing the vehicle/survivor assignment in the description of the heuristics. But it can also be interesting to evaluate the quality of the final solution if we take the minimum merit of the vehicle/survivor assignment.

3.3.4 Variants obtained by combining different criteria

Using the schemes of heuristics HC1 and HC2 and combining different criteria, several heuristics can be obtained for the MMRO problem. The next figure resumes several possible heuristics and their interpretation:

Table 2. Possible variations for the constructive heuristics

Heuristic variant	Heuristic scheme	Criteria for vehicle/survior assignment	Survivor related health information	Priority rule for choosing survivor
HC1d	HC1	Distance	Not available	-
HC1e	HC1	ETA	Not available	-
HC1p	HC1	Profit	Available	-
HC2d	HC2	Distance	Not available	Distance
HC2e	HC2	ETA	Not available	ETA
HC2p	HC2	profit	available	Remaining lifetime

Heuristics HC1d and HC1e are the ones that best reproduces the rescue procedures that are followed by the rescue units dispatched to the scene in a real SAR operation. The rule for choosing the survivor to be recovered is usually defined by the expected time arrival (ETA) to the location of the survivor. If there is only one vehicle or if all vehicles have the same speed then the ETA criteria matches the distance criteria. This is related with the distance between the location of the vehicles and the location of the survivor and also the speed of the vehicle and the weather conditions on the scene.

In a hypothetical situation where the SAR coordinator know with absolute certain the maritime drift of the survivors and do not have the optimal rescue plan, one may ask what rescue procedure will return the best results. In this situation, the availability of health information regarding survivors will increase the number of possible rescue procedures or heuristics to consider. Considering such a possibility, the question we would like to answer is: “if health information regarding survivors is known *a priori* what is the best rescue procedure to adopt?" If health information is available before the rescue operations we may be tempted to presume that we have an advantage in order to implement a rescue plan that minimizes loss of life. But this is only true if we have the optimal plan or optimal solution for the MMRO rescue plan. This plan or rescue solution is usually not available prior to the rescue operations. Only the "rescue procedure" or "rescue heuristic" is available and that will hardly guarantee the optimal rescue plan.

3.4 Pilot method

Exact methods to solve large instances of the MMRO problem have performed poorly when the number of SAR objects is greater than fifty. In order to obtain better quality solutions for larger instances of the MMRO problem we present a pilot method that uses the constructive heuristics in the previous Section as the pilot heuristic (or sub heuristic). The LP formulation of the MMRO problem implies a large number of variables and constraints for a medium-size incident, even with a few dozen survivors. We have solved instances with 40 survivors, 144 time stamps (cluster size) and 3 vehicles with exact methods based on the LP formulation at the cost of several days of CPU time. For the same problem instance, the constructive heuristics can obtain a feasible solution in a fraction of seconds. This is quite encouraging if one is interested in testing a repetition algorithm within a pilot scheme with the presented constructive heuristics acting as the pilot heuristic.

For the MMRO problem the pilot method builds a partial solution in each iteration called the "master solution" M. Since the partial solution is built by adding a feasible vehicle/survivor assignment, one has to decide the criteria to choose such assignment. In the previous Section, several criteria where presented to choose such an assignment for the constructive heuristics and those are quite inexpensive to evaluate. Let A_M be the set of all feasible vehicle/survivor assignments that can be added to the partial solution M. For all feasible vehicle/survivor assignment $a \in A_M$, one can obtain a complete and feasible solution for the MMRO problem by using another efficient heuristic (called pilot heuristic or sub heuristic) to extend the partial solution M with the assignment a included. Let $p(a)$ denote the objective value of the solution obtained by the pilot heuristic for all $a \in A_M$ and let a_0 be the most promising feasible vehicle/survivor assignment according to the sub heuristic, that is, $p(a_0) \leq p(a)$ for all $a \in A_M$. The feasible vehicle/survivor assignment a_0 is added to the master solution M. New pilot calculations can be performed from the changed master solution M until further pilot calculations do not lead to improvement or the master solution is complete (master solution M is complete when the set A_M is empty). Since the master solution is only feasible when all the vehicles return to a depot, it is necessary to guarantee that all vehicles end their tour at a depot when no more feasible vehicle/survivor assignments can be made.

Evaluating all feasible vehicle/survivor assignments for a given master solution M can be a quite labouring and time consuming process. For instance, if the MMRO problem has k vehicles and n survivors, this implies $k \times n$ calls of the pilot heuristic to choose the most promising assignment. The feasible vehicle/survivor assignments are also described by additional information that can be taken into consideration if we are interested to choose a subset of assignments from the set A_M. For example, one can choose, for each vehicle, the m feasible assignments (one assignment corresponds to one SAR object) that have the lowest arrival time to the location of the survivor from the current location of the vehicle (one could also consider the shortest distance or the profit). This selection can be based on the "merit" function previously defined in the constructive heuristics. In this case the pilot calculation implies calling the pilot heuristic to evaluate $k \times m$ assignments (where $n > m$). In the MMRO problem this method to reduce the number of calls of the pilot heuristic may prove to be beneficial because assignments with a high arrival time between vehicle and survivor are not very common in an optimal solution. Since we don't want to exclude vehicles when choosing feasible assignments the limit should be defined on the number of survivors that are going to be evaluated for each vehicle. This number represents an upper limit on the number of feasible assignments to each vehicle since the number of feasible vehicle/survivor assignments for a given vehicle may be smaller than the limit m. Given a master solution M, the set A_M can be represented by the list of the feasible vehicle/survivor assignments that can be made conditional to the solution M (which is conditional to the location of the vehicles and survivors in the partial solution). If we choose to select only the m assignments for each vehicle, then the list should be reduced by supressing the assignments that do not meet the merit criteria and thus we obtain the set $\mathring{A}_M$. It may happen during the algorithm that the set $\mathring{A}_M$ is empty while there are still survivors to be recovered and vehicles whose status hasn't been checked in order to set its correspondent variable $\boldsymbol{rc(k)}$ to zero. Note that the vector $\boldsymbol{rc}$ is associated to a particular partial solution and it is not changed while evaluating future vehicle/survivor assignments.

The next algorithm describes a pilot method where the evaluation of candidate's assignments to be included in the master solution results from selecting the m highest merit assignments using a merit function $\boldsymbol{\varphi}$.

Algorithm 3.4 – PH1 Pilot Heuristic for the MMRO problem (one level branching)

Inputs: Number of Helicopters to employ $\boldsymbol{nh}$, their characteristics (range, passenger capacity), number of nearby ships $\boldsymbol{nh}$ and passenger capacity, distance matrixes $\boldsymbol{D^h}$ and $\boldsymbol{D^{ns}}$ for helicopters and nearby ships; $\boldsymbol{U = nh + ns}$, selection parameter $\boldsymbol{m}$; Merit function $\boldsymbol{\varphi}$.

Output: solution vectors $\boldsymbol{vss}$ and $\boldsymbol{vsk}$ for each vehicle, solution cost

Initialize vectors $\boldsymbol{vss}$ and $\boldsymbol{vsk}$ with the initial starting node for each vehicle (master solution $\boldsymbol{M}$);
While $|\mathring{A}_M| > 0$ *and* $|S| > 0$ *and* $\sum_{k=1}^{U} \boldsymbol{rc}(k) > 0$
    For $k = 1{:}U$
        For all $s \in S$, Calculate $\varphi(k, s)$;
            If $\varphi(k, s) = \emptyset \; \forall \; s \in S$, then $\boldsymbol{rc}(k) = 0$; End if
    Create $\mathring{A}_M$ by selecting for each vehicle the $\boldsymbol{m}$ feasible assignments with highest merit
    If $|\mathring{A}_M| > 0$,
        For all $a \in \mathring{A}_M$
            Calculate profit $p(a)$ with pilot heuristic starting from solution M with $\boldsymbol{a}$ included
        $a_0 = argmax\{p(a), a \in \mathring{A}_M\}$;
        $(k^*, q^*) = \max_{k=1,\ldots U, q \in S_{a_0}} \left\{p\left(k_{a_0}, s_{a_0}\right): D^{k_{a_0}}\left(vss\{k_{a_0}\}(end), f\left(s_{a_0}, q\right)\right) > 0\right\}$
        $vss\{k^*\} = [vss\{k^*\}(end)\ q^*]$; $vsk\{k^*\} = [vsk\{k^*\}\ s_{a_0}]$; $S \leftarrow S \backslash \{s_{a_0}\}$
    End If
For $k = 1{:}U$
    Examine vehicle k and terminate route if $\boldsymbol{rc}(k) = 0$ or $|S| = 0$
Calculate cost of solution and return $\boldsymbol{vss}$ and $\boldsymbol{vsk}$

Comparing the values $p(a)$ and $\varphi(a)$ within the pilot method it may happen that the assignment with the highest merit may not be the most promising according to the objective value of the pilot heuristic. By selecting the most promising assignment instead of the one with the highest merit the algorithm is avoiding potential "bad" decisions.

The process of "looking" for better decisions (which are associated to a specific assignment during the pilot method) can be improved if, instead of evaluating one feasible assignment among all possible assignments associated to a partial solution, the procedure would evaluate sequences of assignments. This implies that the algorithm would "branch" feasible assignments on a first level even further depending on the desired sequence length. If we evaluate sequences of two feasible assignments (with the second assignment being conditional to the first one) we would have an "assignment tree" with two levels where in the first level we would enumerate all assignments associated within $\mathring{A}_M$ and in the second level we would enumerate all the feasible vehicle/survivor assignments for each element in $\mathring{A}_M$. Once again the total number of possible sequences of feasible assignments would be quite large. For this reason it would be prudent to limit the number of feasible assignments that should be considered in the 2nd level. When including a sequence of assignments to a master solution the objective value of the extended solution obtained with the pilot heuristic can be used to characterize the first assignment of the sequence or the sequence itself. One can include the first assignment or the sequence of assignments to the master solution. This last option will increase the speed of the pilot method since in each iteration the partial solution is extended with a number of assignments depending of the total number of assignments in the sequence. The consequence of this procedure to build the partial solution is that we may be missing along the procedure other more promising assignments that lead to better solutions.

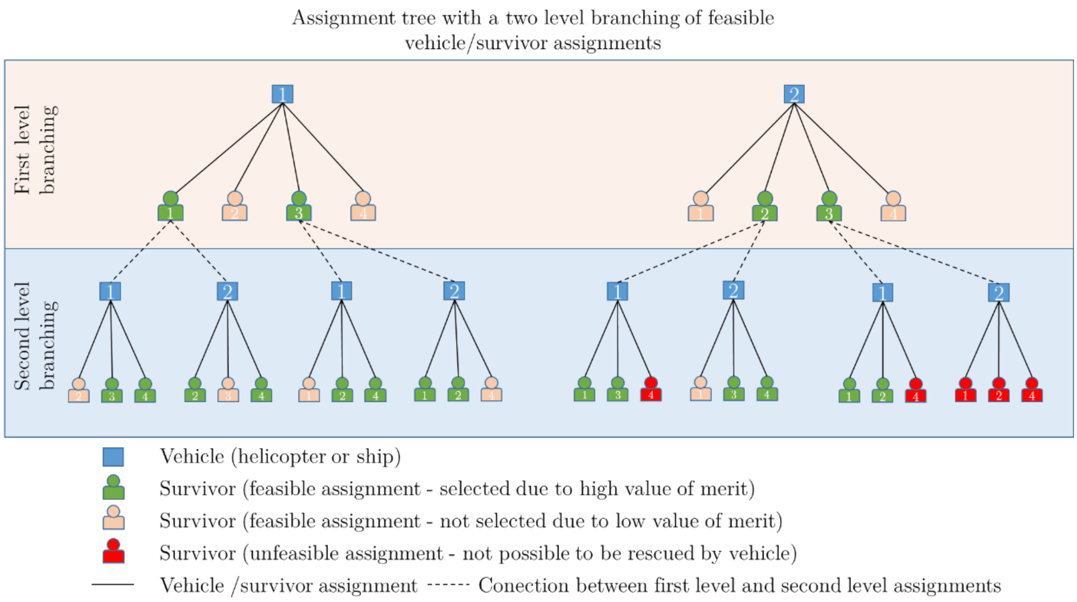

Figure 24. Assignment tree for enumerating sequences of two successive feasible assignments

Figure 24 illustrates an “assignment tree” with a two level branching of feasible assignments corresponding to sequences of two assignments associated to a partial solution M and applying a limit of considering only the two survivors for each vehicle with the most merit within each level ($m = 2$). The lines connecting the blue squares with the green symbols represent feasible vehicle/survivor assignments. The dashed lines combines these feasible assignments and denote sequences of two consecutive assignments at a given iteration of the pilot method.

It may occur that a second level assignment may not be feasible due to the circumstances of the rescue operation (for example, not enough autonomy to recover survivor or the survivor is too far away to be rescued within the mission time period). In the above figure we can observe that vehicle 2 cannot rescue survivor 1, 2 and 4 after rescuing survivor 3. This means that if we are interested to consider sequences of assignments to include in the partial solution and check the objective value with the pilot heuristic, then it is important to check the feasibility of the succeeding assignments. Another important aspect of the pilot method is that the pilot heuristic has to process $n-k$ survivors in iteration k (because in each iteration one survivor is recovered). This makes the last iterations of the pilot method less time consuming than the first ones.

The branching of feasible vehicle/survivor assignments can be made up to several levels until no more branching is possible. If the problem has n survivors then the maximum number of levels (equivalent to maximum length of sequences of assignments) is also n. Such enumeration is only possible in the first iteration where the partial solution does not have any assignment. Naturally, if one would consider all possible sequences of assignments we would eventually enumerate all possible solutions of the MMRO problem in the first iteration of the pilot method. Adding a chosen assignment at iteration k in algorithm 3.4 eliminates the study of $n-k-1$ possible ways to evaluate potencial feasible solutions. Note that if we choose (see Figure 24) an assignment $a_0 \in A_M$ to include in the partial solution belonging to the first level it implies that in the next iteration all the sequences that would have begun in all the other assignments $a \in A_M \setminus \{a_0\}$ at the first level won't be studied.

The implementation of the pilot method implies the selection of several parameters and features which will lead to different heuristic schemes and subsequently to different solutions. For the MMRO problem, the key features to obtain different variants of the pilot method are:

- Maximum number of feasible assignments evaluated by the pilot heuristic in each iteration of the pilot method.
- Sequence of vehicle/survivor assignment length (this parameter can also be stated has the depth or the number of levels in the assignment tree).
- Number of assignments to be added to the partial solution.
- The pilot heuristic used within the pilot method.
- Merit function used to choose select the assignments to be evaluated by the pilot heuristic.

Some of the features depicted above can be set as parameters of the pilot method. For example the maximum number of feasible assignments evaluated in each level can be represented by a corresponding variable. As for the depth of the assignment tree, it may not be so obvious how to implement such feature as a parameter in the pilot method. The challenge in implementing a pilot method with h levels involves investigating the feasibility of the sequences of assignments. Note that it may not be possible to have sequences of length h (some of the sequences may have a length smaller than h due to the feasibility of their respective assignments). The next algorithm depicts a pilot method for the MMRO problem with 2 levels.

Algorithm 3.5 – PH2 Pilot Heuristic for the MMRO problem (Two level branching)

Inputs: Number of Helicopters to employ $\boldsymbol{nh}$, their characteristics (range, passenger capacity), number of nearby ships $\boldsymbol{nh}$ and passenger capacity, distance matrixes $\boldsymbol{D^h}$ and $\boldsymbol{D^{ns}}$ for helicopters and nearby ships; $\boldsymbol{U = nh + ns}$, selection parameter $\boldsymbol{m_1}$ and $\boldsymbol{m_2}$; Merit function $\boldsymbol{\varphi}$.

Output: solution vectors $\boldsymbol{vss}$ and $\boldsymbol{vsk}$ for each vehicle, solution cost

1: Initialize vectors $\boldsymbol{vss}$ and $\boldsymbol{vsk}$ with the initial starting node for each vehicle (master solution $\boldsymbol{M}$);
2: While $|\mathring{A}_M| > 0$ *and* $|S| > 0$ *and* $\sum_{k=1}^{U} \boldsymbol{rc}(k) > 0$
3: For $k = 1{:}U$
4: For all $s \in S$, Calculate $\varphi(k, s)$;
5: If $\varphi(k, s) = \emptyset \; \forall \; s \in S$, then $\boldsymbol{rc}(k) = 0$; End if
6: Create $\mathring{A}_M$ by calling **procedure LSEQ** with parameters $\boldsymbol{m_1}$ and $\boldsymbol{m_2}$
7: If $|\mathring{A}_M| > 0$
8: For all $\pi \in \mathring{A}_M$
9 Calculate profit $p(\pi)$ with pilot heuristic starting from solution M with π included
10: $\pi_0 = argmax\{p(\pi), \pi \in A_M\}$; Take a_0 as the first level assignment in sequence π_0
11: $(k^*, q^*) = \max_{k=1,\dots U, q \in S_{a_0}} \left\{p\left(k_{a_0}, s_{a_0}\right) : D^{k_{a_0}}\left(vss\{k_{a_0}\}(end), f\left(s_{a_0}, q\right)\right) > 0\right\}$
12: $vss\{k^*\} = [vss\{k^*\}(end)\; q^*]$; $vsk\{k^*\} = [vsk\{k^*\}\; s_{a_0}]$; $S \leftarrow S \backslash \{s_{a_0}\}$
13: End If
14: For $k = 1{:}U$
15: Examine vehicle k and terminate route if $\boldsymbol{rc}(k) = 0$ or $|S| = 0$
16: Calculate cost of solution and return $\boldsymbol{vss}$ and $\boldsymbol{vsk}$

In algorithm 3.5 the sequence of assignments π may not necessarily have two consecutives assignments. The sequence π may have a single feasible vehicle/survivor assignment. The list of the sequences $\pi \in A_M$ has to be calculated taking into consideration the master solution at the present iteration of the pilot method. The listing of all feasible sequences is done with the procedure **LSEQ**.

Procedure 1 – LSEQ Sequencing of feasible assignments from a partial solution M

Inputs: Partial solution $\boldsymbol{M}$; parameters $\boldsymbol{m_1}$ and $\boldsymbol{m_2}$; Merit function $\boldsymbol{\varphi}$; Set of survivors S.

Output: List of sequences of feasible assignments $\boldsymbol{A_M}$

1: $S' = S$ (S' is a temporary set of remaining survivors)
2: $A_M = \emptyset$ (The list of sequences is empty)
3: For $k_1 = 1: U$
4:     Calculate $W(k_1)$ ($W(k_1)$is the set of all feasible assignments for vehicle k_1)
5:     Reduce $W(k_1)$ to contain only its $\boldsymbol{m_1}$ highest merit assignments
6:     If $|W(k_1)| > 0$
7:         For all $j_1 \in W(k_1)$
8:             $S' = S' \backslash \{s(j_1)\}$; (remove survivor $s(j_1)$ from set of remaining survivors)
9:             $\boldsymbol{M_1} = \boldsymbol{M} \cup \{j_1\}$; (add assignment j_1 to partial solution $\boldsymbol{M}$)
10:             If $|S'| > 0$
11:                 For $k_2 = 1: U$
12:                     Calculate $W(k_2)$ from partial solution $\boldsymbol{M_1}$
13:                     Reduce $W(k_2)$ to contain only its $\boldsymbol{m_2}$ highest merit assignments
14:                     If $|W(k_2)| > 0$
15:                         For all $j_2 \in W(k_2)$
16:                             $A_M = A_M \cup (j_1, j_2)$
17:                     else
18:                         $A_M = A_M \cup j_1$
19:                     End If
20:             else
21:                 $A_M = A_M \cup j_1$
22:             End If
23:     End If
24: Return A_M

In the above procedure one can observe that listing the sequences of assignments from a partial solution M requires checking two important conditions:

- Whether or not a vehicle can visit a client from its current location within the partial solution. This corresponds to check if the set $W(k)$ is not empty for a vehicle k (line 6 and 14 of the procedure).
- When moving from a level to the next one it is necessary to check if all survivors have been rescued. This is needed because if there are no more survivors to

rescue then there is no need to consider any further assignments (line 10 of the procedure). This condition is not needed in the last level of the assignment tree.

Both conditions are important to guarantee that all the sequences of assignments will lead to feasible solutions of the MMRO problem and they have to be checked at each level of the assignment tree (with the exception for the second condition in the last level of the assignment tree). The procedure LSEQ can be generalized to any number of levels. Nonetheless the number of levels cannot exceed the number of survivors or objects to recover.

3.4.1 Enumeration of all possible solutions with the pilot method

It is also possible to perform a complete enumeration of all solutions of the MMRO problem with the pilot method. This can be done having as much levels in the assignment tree as the number of objects to recover and having no limit in the number of feasible assignments evaluated at each level. Most of the sequences of assignments wouldn't be feasible but the process would enumerate all feasible solutions. If we impose no limits on the number of assignments to be evaluated on each level and if the number of levels in the assignment tree is equal to the number of survivors or objects to recover, then in the first iteration of the pilot method the set A_M (A_M is built from the master solution that only has the initial depot for each vehicle) contains all the possible solutions for the MMRO problem. In this later case, the sequences of assignments in the set A_M will represent almost fully grown solutions. These sequences only require that the vehicles return to a depot to finish their tour. Since there is no cost associated to travelling between a datum and a depot it doesn't make any difference the choice for the ending depot for each vehicle.

The pilot method with such setting would only need to perform its first iteration. This is illustrated in Figure 25 with an example of 2 vehicles and 3 survivors.

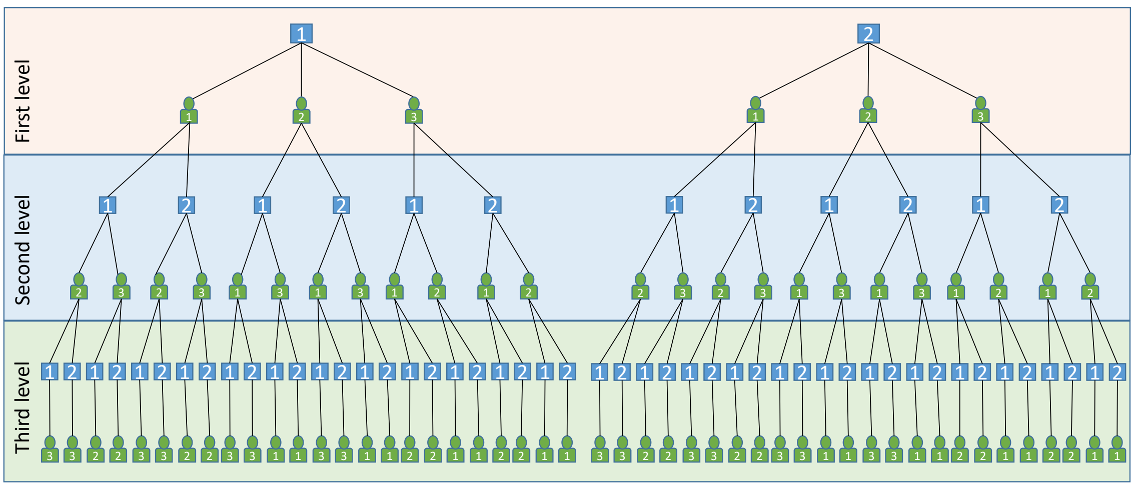

Figure 25. Enumeration of solutions for MMRO problem with 2 vehicles and 3 objects to recover

The sequencing procedure may produce sequences that, once added to their partial solution, will lead to the same master solution. This can be observed in the above figure where the sequences of assignments represent fully grown solutions of the MMRO problem. For example, the sequence $\big((1,1),(2,2),(2,3)\big)$ codes the same solution as the sequence $\big((2,2),(1,1),(2,3)\big)$. This is due to the fact that vehicle 2 goes from its starting depot to the datum of object 2 and afterwards heads towards the location of object 3. The time stamps where vehicle 2 visits objects 2 and 3 are the same in both sequences, since the operation between vehicles is independent from each other. Another way to observe this redundancy is that the sequence of objects visited by each vehicle is the same on both sequences.

The maximum number of feasible assignments evaluated for each master solution and the sequence length of feasible assignments are two key parameters that will greatly affect the speed of the pilot method. Both parameters determine the total number of evaluations performed by the pilot heuristic in each iteration of the pilot method. If we consider the pilot method with L levels and e_l the maximum number of assignments evaluated in the $l-level$ (with $L<n$) for each vehicle, then the total number of evaluations at the beginning of iteration j (partial solution has $j-1$ assignments) is $k^l \prod_{l=1}^{L} e_l$. For example, in iteration 1 of a pilot method with 2 levels and considering a limit of 4 assignments on level 1 and 6 assignments on level 2 the pilot method will evaluate $24k^2$ assignments. With 5 vehicles there would be 600 calls of the pilot heuristic. The number of calls would decrease and their total time would also decrease since in each iteration of the pilot method there is one less survivor to take into consideration by the pilot heuristic.

3.5 Prototype for building MMRO instances

A graphical user interface (GUI) was developed in MATLAB to create instances of the MMRO problem. The GUI is a prototype of a decision support system which can also be used as a laboratory to test algorithms and other mathematical models for the MMRO problem. In this dissertation, this GUI is designated as MMRO Design Tool (MDT) and is categorized as a prototype, as it is intended to be a vehicle for demonstration of several functionalities that can be implemented in commercial Decision Support Systems (DSS), such as the Oversee. The MDT prototype allows the analyst to create instances, find solutions with different heuristics and compare results regarding their performance as well as the performance of different heuristics. The analyst can also save the problem data into a file and append the solution obtained with different heuristics to it.

The MDT prototype provides access to several other interfaces that were developed for different analysis purposes. For example, the user may be interested to obtain other performance indicators associated with a solution. Or, it may require to compare the performance of different variants of the pilot method for the same problem. In this Section we summarize the key functionalities available in the MDT prototype.

Since this work is intended to conduct studies aimed at the Portuguese maritime areas the map points directly at the Portuguese SRR and the white circle shows the Montijo Airbase where the Search and Rescue helicopters are stationed and also Lisbon Figo Maduro airport.

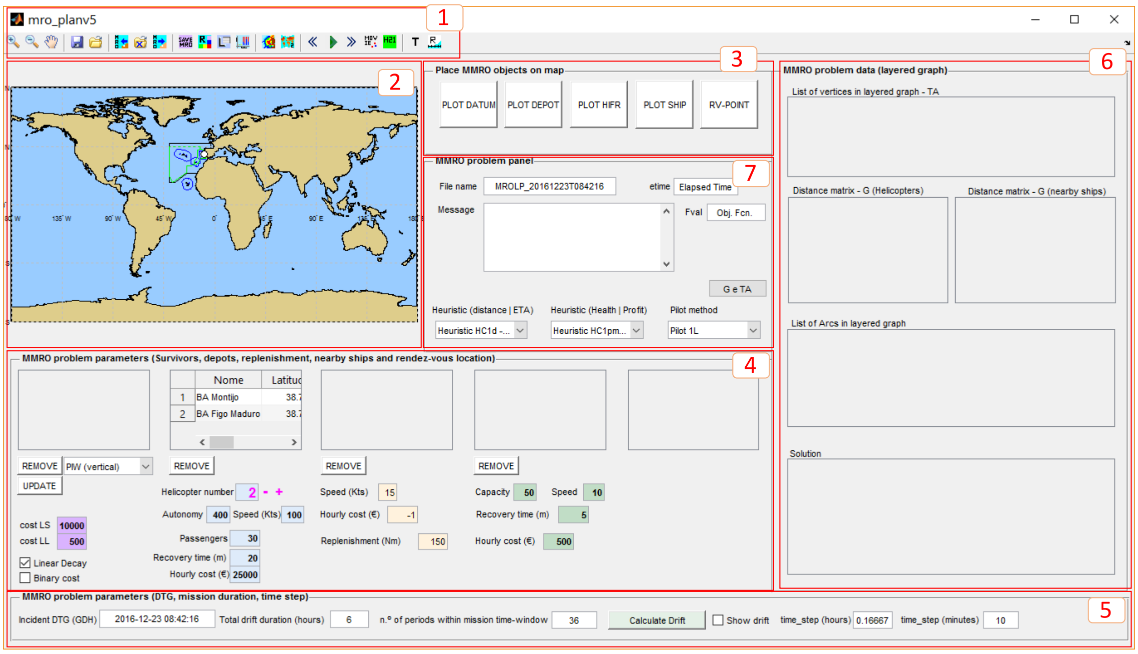

Figure 26. Prototype main interface to build and analyse MMRO problems

The prototype interface is divided into several panels. Figure 26 shows the main interface of the MDT prototype, which is grouped into 7 different panels:

Panel 1. Comprises a toolbar with several buttons grouped according to their function (a separator is used to separate buttons from different sets). The first set of three buttons (from left to right) allow the user to zoom in and out and pan the map. The second set allows the user to save and load an MMRO problem. The third set allows the user to add or retrieve a solution to a problem. The fourth set allows the user to build the MMRO problem data and saved it into a file and access to a dashboard that shows the efficacy of the MMRO solution. The remaining buttons give access to other interfaces where the user can observe the solution on the layered graph, compare different solutions obtained by different variations of the pilot method and playback the movie of the rescue operation for a given MMRO solution.

Panel 2. Map axes. This is where the user can manually set the location of objects to recover and the location of nearby ships and airports. It also shows the MMRO solution.

Panel 3. MMRO problem objects. This panel has five buttons to each one of the category of objects in the MMRO problem.

Panel 4. Description of the each object in the MMRO instance. This panel presents five tables that list each object of each category. The first table list each object whether it's a single person in the water or a liferaft with several survivors. The second table list the available airports for helicopter to start and end their tour. The third table lists available replenishment ships. The fourth table lists available nearby ships and the last table list the meeting location for passenger's transhipment.

Panel 5. This panel allows the user to set the initial alert date-time group (DTG) that references the start of the rescue operation. It also allows the user to choose the mission duration or mission time window and the number of time stamps within that duration. It also allows the user to calculate the maritime drift for the objects in the water.

Panel 6. This panel shows several tables related with the data structures of the MMRO problem. On the top, the first table shows a list with the description of the nodes of the layered graph. Bellow there are two tables that show the distance matrixes for helicopters and nearby ships. The fourth table (counting from top to bottom) lists several variables that describe the arcs in the layered graph. The variables describing each arc comprise the indexes of the arcs, the indexes (of object and time stamp) of the starting and ending node, distance, associated vehicle and profit. The last table shows the subset of arcs that belong to the solution being presented.

Panel 7. This panel allows the user to set the name of the file containing all the data structures and solutions of the MMRO problem. It also shows the description of solution being displayed, the CPU time it took to achieve it and its objective function value. In this panel, the user can also select several heuristics along with the pilot method to find a solution for the MMRO problem.

3.5.1 Building an instance

Consider a small example of a MMRO problem with two vehicles (one helicopter and one nearby ship) and five survivors, all of them in the water. The survivors are located southwest of Lisbon's city and the farthest survivor to be recovered is approximately 80 nautical miles due west from Montijo Airbase.

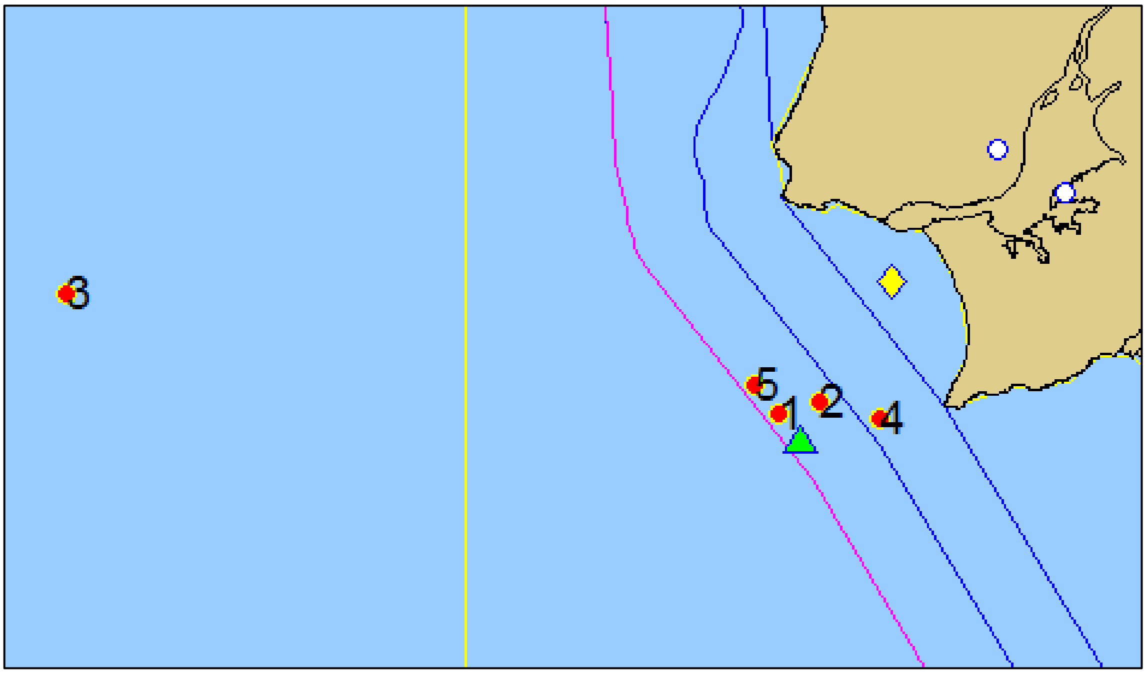

Figure 27. Initial location of distress survivors that require recovery

The problem's incident has the following data:

- Incident DTG: 2016-12-26 08:19:59
- Total drift duration (mission time-window): 3 hours
- Time step: 5 minutes
- Number of time stamps: 36 (3 hours has 36 periods of 5 minutes each)
- Available vehicles:
  - 1 helicopter located at Montijo Airbase
    - Cruise speed: 100 Kts
    - Time to recover one survivor: 20 minutes
    - Autonomy: 400 nautical miles
  - 1 nearby ship located at coordinates 38.3611 latitude and -9.4570 longitude
    - Cruise speed: 10 Kts
    - Time to recover one survivor: 5 minutes
    - Autonomy: Inf
- Available airbases:
  - Montijo Airbase
    - located at coordinates 38.7133 latitude and -9.0260 longitude
  - Figo Maduro Airbase

- located at coordinates 38.7756 latitude and -9.1358 longitude

- Meeting location for survivors transfer at coordinates 38.5894 latitude and -9.3083 longitude
- SAR Object type: Person in water (PIW)
- Survivors initial location and estimated time alive:

Table 3. Location of PIW in MMRO instance.

Survivor	Longitude	Latitude	RED DTG
1	38.4021	-9.4908	2016-12-26 08:40:11
2	38.4196	-9.4232	2016-12-26 10:17:11
3	38.5718	-10.6533	2016-12-26 09:08:40
4	38.3962	-9.3286	2016-12-26 10:34:50
5	38.4431	-9.5313	2016-12-26 09:10:26

  - The estimated time alive was generated from a uniform distribution between the initial DTG and the mission time-window. The expected time of death corresponds to the "RED DTG" column. From the above table it's possible to observe that survivor 1 has the least time to survivor. The user can also define the RED DTG by changing its value in the survivors table in panel 4.

- Survivor's drift
  - The drift is calculated using wind data from GRIB files. The figure bellow shows the location of each object estimated by the previous drift model presented at each one of the 36 time stamps ranging from the initial DTG and 3 hours later:

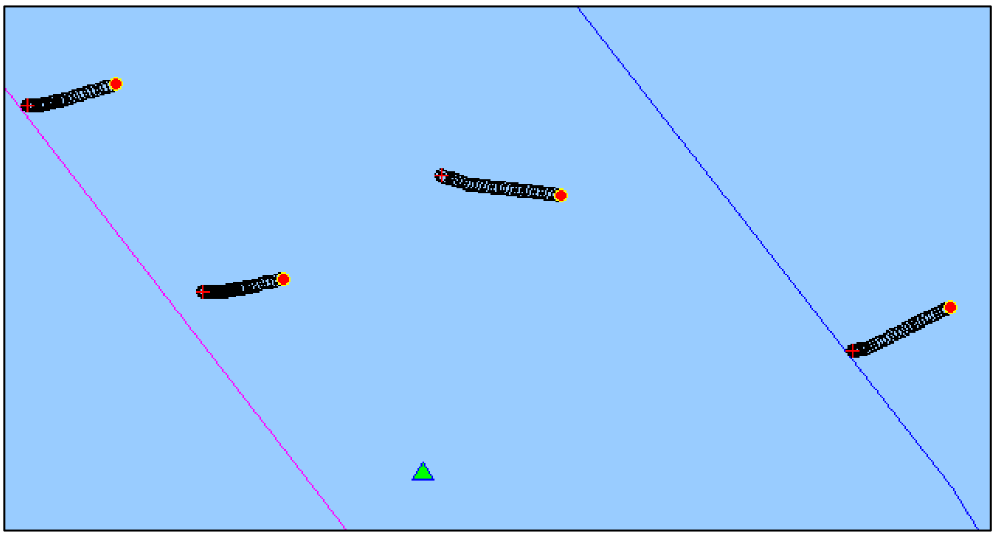

Figure 28. Survivors drift along 3 hours within water

From the "incident's data" it is possible to build the MMRO problem data structures associated with the layered graph G. The first step is to build the distance matrixes for helicopters and nearby ships taking into consideration their average cruising speed. The distance matrixes represents the distance between two consecutive datum and these correspond to nodes in our layered graph. Relevant information associated to the nodes (or datum) are kept in a relational table. Since each node is time-indexed we know a priori the location and the expected time to survive at each time stamp. For this small example, the layered graph will have 184 nodes. In the figure bellow we present the first 20 nodes of the layered graph G:

mro_ver_tabelaTA

Tabela de associação - TA

	Nº Vértice	Nº do obj (K)	t	Latitude	Longitude	GDH	Custo	Tipo de datum
1	1	1	1	38.7133	-9.0260	7.3669e+05	0	1
2	2	2	1	38.7756	-9.1358	7.3669e+05	0	1
3	3	3	1	38.3611	-9.4570	7.3669e+05	0	2
4	4	4	1	38.5894	-9.3083	7.3669e+05	0	3
5	5	5	1	38.4021	-9.4908	7.3669e+05	9500	5
6	6	5	2	38.4019	-9.4917	7.3669e+05	8769	5
7	7	5	3	38.4017	-9.4928	7.3669e+05	8038	5
8	8	5	4	38.4015	-9.4938	7.3669e+05	7307	5
9	9	5	5	38.4014	-9.4944	7.3669e+05	6576	5
10	10	5	6	38.4012	-9.4953	7.3669e+05	5845	5
11	11	5	7	38.4010	-9.4963	7.3669e+05	5114	5
12	12	5	8	38.4008	-9.4973	7.3669e+05	4383	5
13	13	5	9	38.4006	-9.4980	7.3669e+05	3652	5
14	14	5	10	38.4005	-9.4987	7.3669e+05	2921	5
15	15	5	11	38.4003	-9.4998	7.3669e+05	2190	5
16	16	5	12	38.4002	-9.5002	7.3669e+05	1459	5
17	17	5	13	38.4002	-9.5006	7.3669e+05	728	5
18	18	5	14	38.4001	-9.5010	7.3669e+05	500	5
19	19	5	15	38.4000	-9.5014	7.3669e+05	500	5
20	20	5	16	38.4000	-9.5018	7.3669e+05	500	5

Figure 29. Node table. Relates node index with location, time, survivability, MMRO object type

The time stamp for each node is coded using MATLAB's *datenum* function where a serial number is used to code a specific date starting from a reference date. The profit for visiting a node is built using a linear decay function starting from the initial incident's DTG and ending at the survivor's expected time of death. The profit values associated to these two moments in time constitute a mere reference. For the present instance we considered a profit value of 10.000 units for the maximum profit associated to recovering a live survivor and a profit of 500 for recovering a deceased survivor. The relation between 10000 and 500 would imply that rescuing more than 20 deceased persons would be more profitable than rescuing one live person. But since the problem only has 5 survivors, the profit for rescuing someone alive will be always greater than rescue several deceased survivors. The data in the node table is crucial for building the helicopters and nearby ships distance matrixes. The figure below shows the helicopter distance matrix in a data table and using MATLAB's *spy* function:

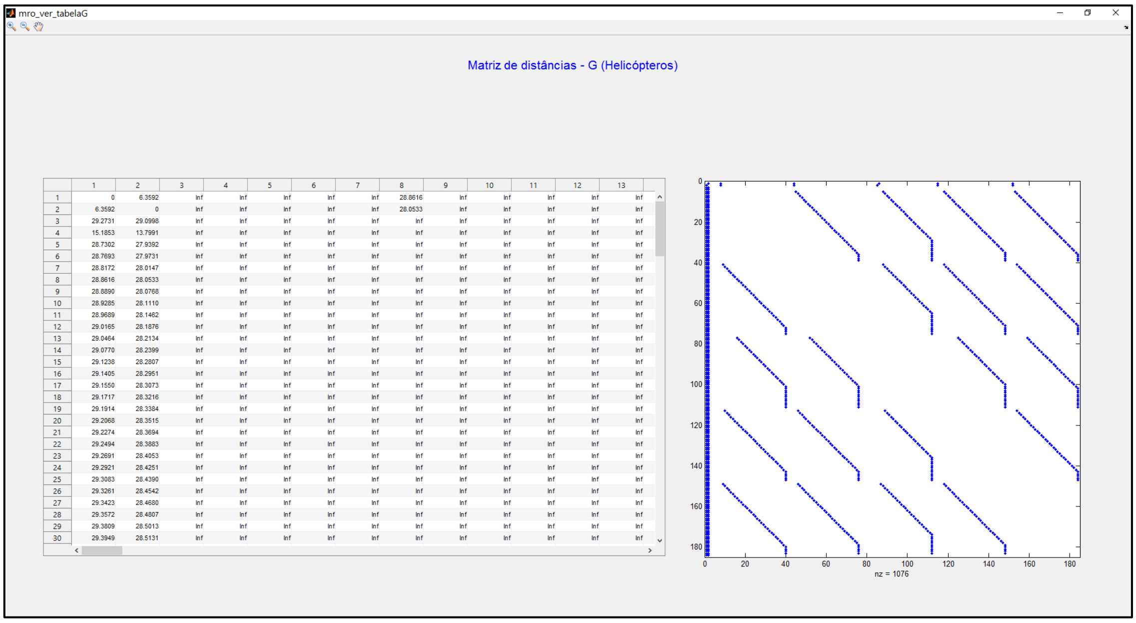

Figure 30. Helicopter's distance matrix

From the above figure it is possible to observe that the helicopter can recover all of the 5 survivors and return to one of the 2 available airbases.

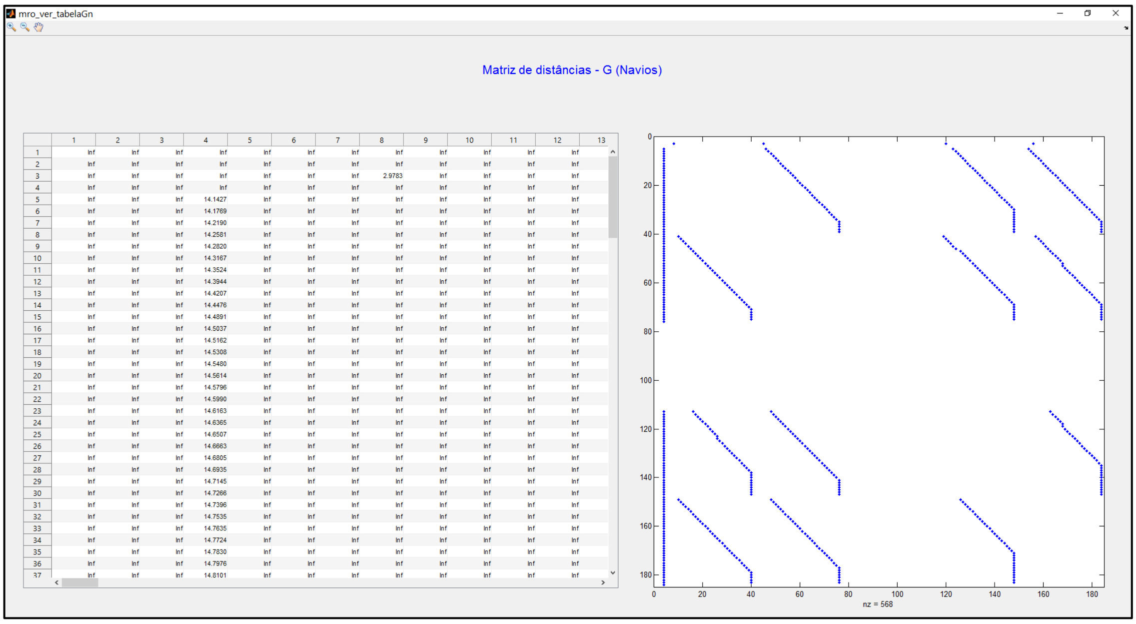

Figure 31. Nearby ship distance matrix

The distance calculation is performed using MATLAB's *distance* function. The distance calculation for each pair of nodes is the most time consuming task while building the MMRO data structures.

In the nearby ship distance matrix it is possible to observe that survivor number 3 cannot be rescued by this vehicle. The reason is that the nearby ships is too far away from the location of survivor number 3 (approximately 80 nautical miles) and at a speed of 10 Kts it is not possible to travel that distance during the mission time-window (3 hours). Notice that the nodes associated to the datum of survivor 3 (columns and rows with index ranging from 77 and 112) have the value "Inf" to all other nodes. Using the above information it is possible to view the nodes of the final layered graph G:

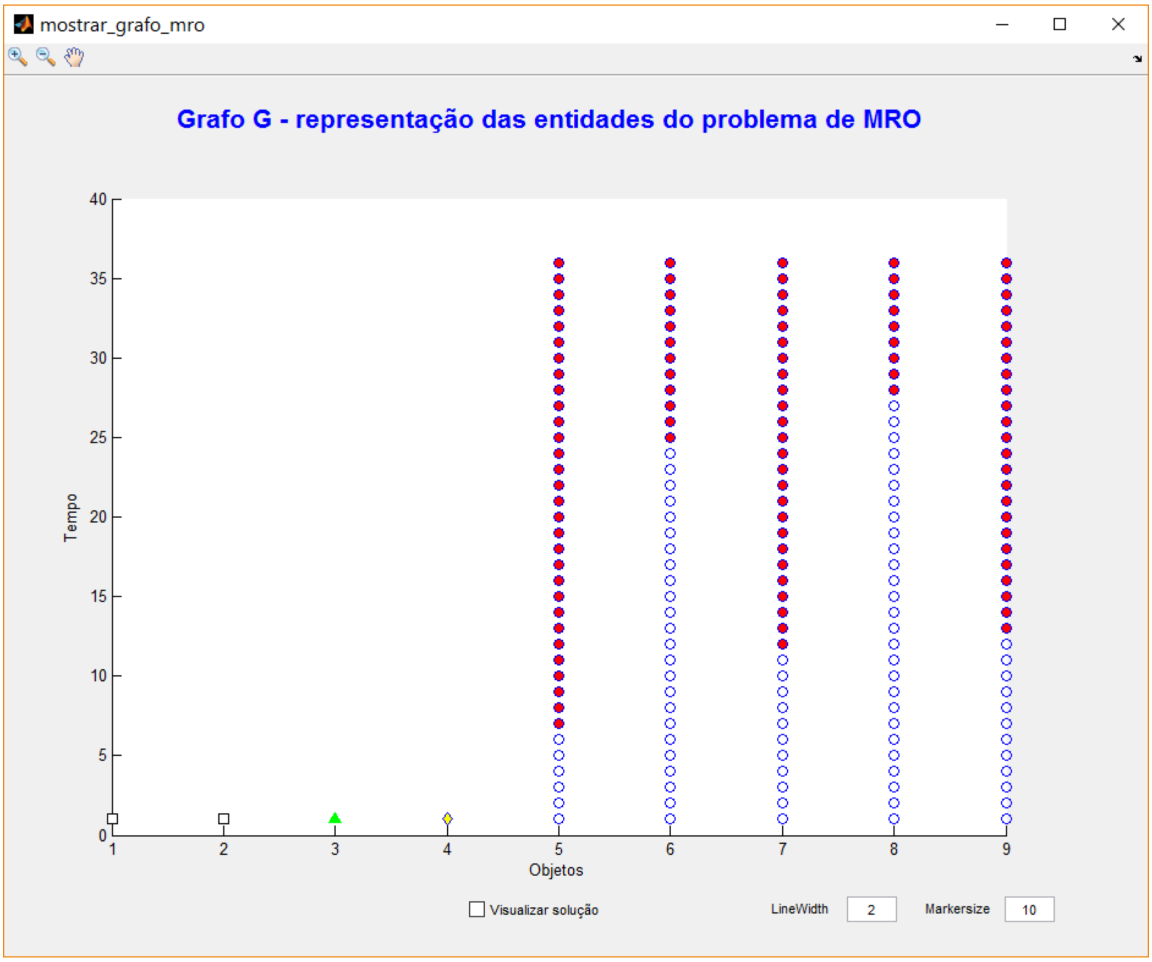

Figure 32. Nodes of Layered Graph within an Object index - time stamp axis

As mentioned before the objects associated to depots (airbases, the initial location of nearby ships and meeting locations) are not time-indexed. They are represented by their respective symbols with object's index 1, 2, 3 and 4. They represent Montijo airbase, Figo Maduro airbase, initial location of the nearby ship and the meeting location for survivors transfer, respectively. The objects whose index start from number five to number nine represent all 5 survivors. The blue circles represent the time-index nodes for each survivor and the red colour indicates that the survivor is deceased at that time stamp. The objective of the rescue operation is to visit the nodes with the highest profit and, off course, avoiding the red ones (since they have the lowest profit).

The arcs of the layered graph are also listed in a table so that information relating cost, nodes and vehicles can be available for to build the constraints matrixes for the BLP problem and heuristic use. The figure below shows a table that lists all feasible arcs in the problem.

mro_ver_tabelaTA

Lista de Arcos - Arc

	Arco	(i,p)	i	p	(j,q)	j	q	distância	veículo	Custo	i	j	Meio 1-H, 2-N
1	1	1	1	1	8	5	4	28.8616	1	0	1	5	1
2	2	1	1	1	44	6	4	25.7523	1	0	1	5	1
3	3	1	1	1	86	7	10	77.1917	1	0	1	5	1
4	4	1	1	1	115	8	3	23.8222	1	0	1	5	1
5	5	1	1	1	152	9	4	28.8765	1	0	1	5	1
6	6	2	2	1	8	5	4	28.0533	1	0	1	5	1
7	7	2	2	1	44	6	4	25.3325	1	0	1	5	1
8	8	2	2	1	85	7	9	72.5352	1	0	1	5	1
9	9	2	2	1	115	8	3	24.5676	1	0	1	5	1
10	10	2	2	1	152	9	4	27.3858	1	0	1	5	1
11	11	5	5	1	1	1	1	28.7302	1	9500	5	1	1
12	12	5	5	1	2	2	1	27.9392	1	9500	5	1	1
13	13	5	5	1	45	6	5	3.1893	1	9500	5	5	1
14	14	5	5	1	88	7	12	56.0620	1	9500	5	5	1
15	15	5	5	1	118	8	6	7.4878	1	9500	5	5	1
16	16	5	5	1	153	9	5	3.1761	1	9500	5	5	1
17	17	6	5	2	1	1	1	28.7693	1	7916	5	1	1
18	18	6	5	2	2	2	1	27.9731	1	7916	5	1	1
19	19	6	5	2	46	6	6	3.1860	1	7916	5	5	1
20	20	6	5	2	89	7	13	56.0690	1	7916	5	5	1

Figure 33. Arcs table. Relates arc indexes with nodes, vehicles, cost and distance

The binary linear model for the MMRO problem is built using the data in the Arc table. Since we are using MATLAB *intlinprog* function to solve the problem optimally, it is convenient to write the MMRO problem in matrix form:

$$\min_{x} f^T x \text{ subject to } \begin{cases} A \cdot x \leq b \\ A_{eq} \cdot x \leq b_{eq} \\ lb \leq x \leq ub \\ x \in \{0,1\} \end{cases}$$

Thus the MMRO problem is solved as a minimization problem. The decision vector x has the same number of components than the number of arcs. It's fairly easy to construct the objective function profit vector *f*, the constraints matrixes A and A_{eq} and the independent vectors b and b_{eq} . The profit vector *f* can be taken from the Arc table as the column corresponding to the profit. The purpose in having the Arc table is to be able to characterize every arc (or variable) with all the indexes of the entities of the MMRO problem. The indexes k, p, q, i, j in the variable x_{ij}^{kpq} are all explicitly defined for each arc in the problem. Each constraint of the binary linear problem is built by selecting the variables with the indexes that are fixed in the constraint. This implies making a query to the Arc table for each constraint in the problem.

For larger instances the number of variables can grow up to several millions. For this reason, matrixes A and A_{eq} are stored as sparse matrixes. Saving the binary problem data as sparse matrixes causes no inconvenient since *intlinprog* accepts them as inputs.

3.5.2 Solving the problem with heuristics

Once the MMRO instance is created the user can save the problem data in to a file which can be accessed later. It can also add different solutions to the problem obtained by different heuristic or optimal methods. The heuristics that are currently available for use are:

- Simple sequential constructive heuristic
  - Variation 1: distance used for merit assessment (HC1d)
  - Variation 2: ETA used for merit assessment (HC1e)
  - Variation 3: maximum profit for merit assessment (HC1p)
- Greedy sequential constructive heuristic
  - Variation 1: distance used for merit assessment (HC2d)
  - Variation 2: ETA used for merit assessment (HC2e)
  - Variation 3: maximum profit for merit assessment (HC2p)
- Pilot method (one, two, three and four levels in the assignment tree)
  - Variation 1: HC1 as sub heuristic
  - Variation 2: HC2 as sub heuristic

Other heuristics can be added to the prototype interface for results comparison. Up to four levels the length of the sequences of assignments to be added to the master solution are made available. More than four levels would imply a large number of calls to the sub heuristic even for a small number of sequences per vehicle in each level.

For the small example, the HC1 heuristic produced the following solution:

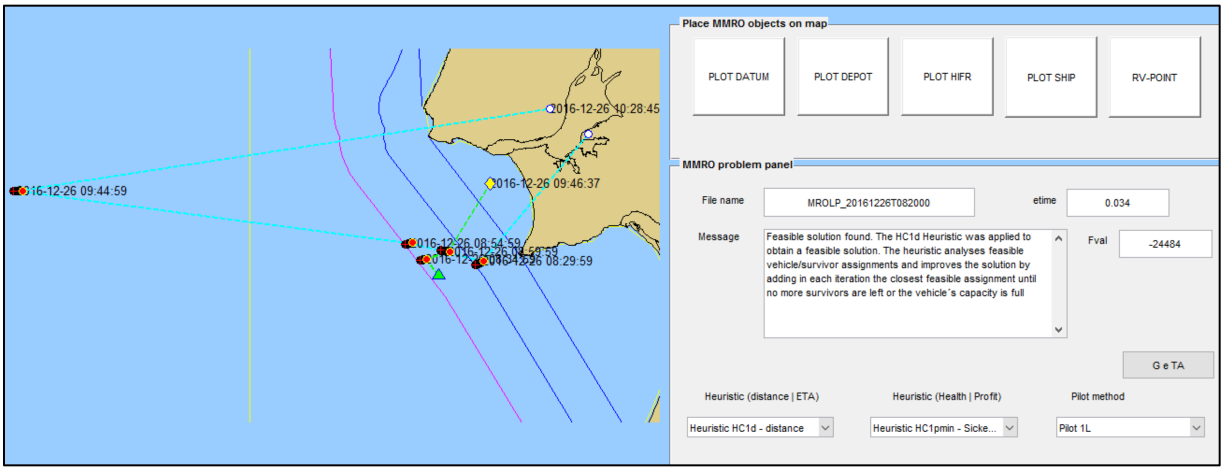

Figure 34. MMRO solution obtained by HC1d

The above solution uses only seven arcs. These can be checked in the table with the solutions arcs:

	Arco	(i,p)	i	p	(j,q)	j	q	distância	veículo	Custo	i	j	Meio 1-H, 2-N
1	4	1	1	1	115	8	3	23.8222	1	0	1	5	1
2	538	94	7	18	2	2	1	72.9608	1	500	5	1	1
3	664	115	8	3	156	9	8	10.1633	1	8820	5	5	1
4	905	156	9	8	94	7	18	53.7459	1	3956	5	5	1
5	1071	3	3	1	8	5	4	2.9783	1	0	2	5	2
6	1088	8	5	4	49	6	9	3.1787	1	4748	5	5	2
7	1248	49	6	9	4	4	1	11.6621	1	6460	5	3	2

Figure 35. Arc description within solution

Figure 35 shows the index of the variables (each variable is associated to an arc in the layered graph) and the indexes of the time-stamps associated to the start and end nodes. The arcs in the table are not ordered according to the sequence of visited objects by each vehicle. The solution can also be viewed in the layered graph G, as depicted in Figure 36:

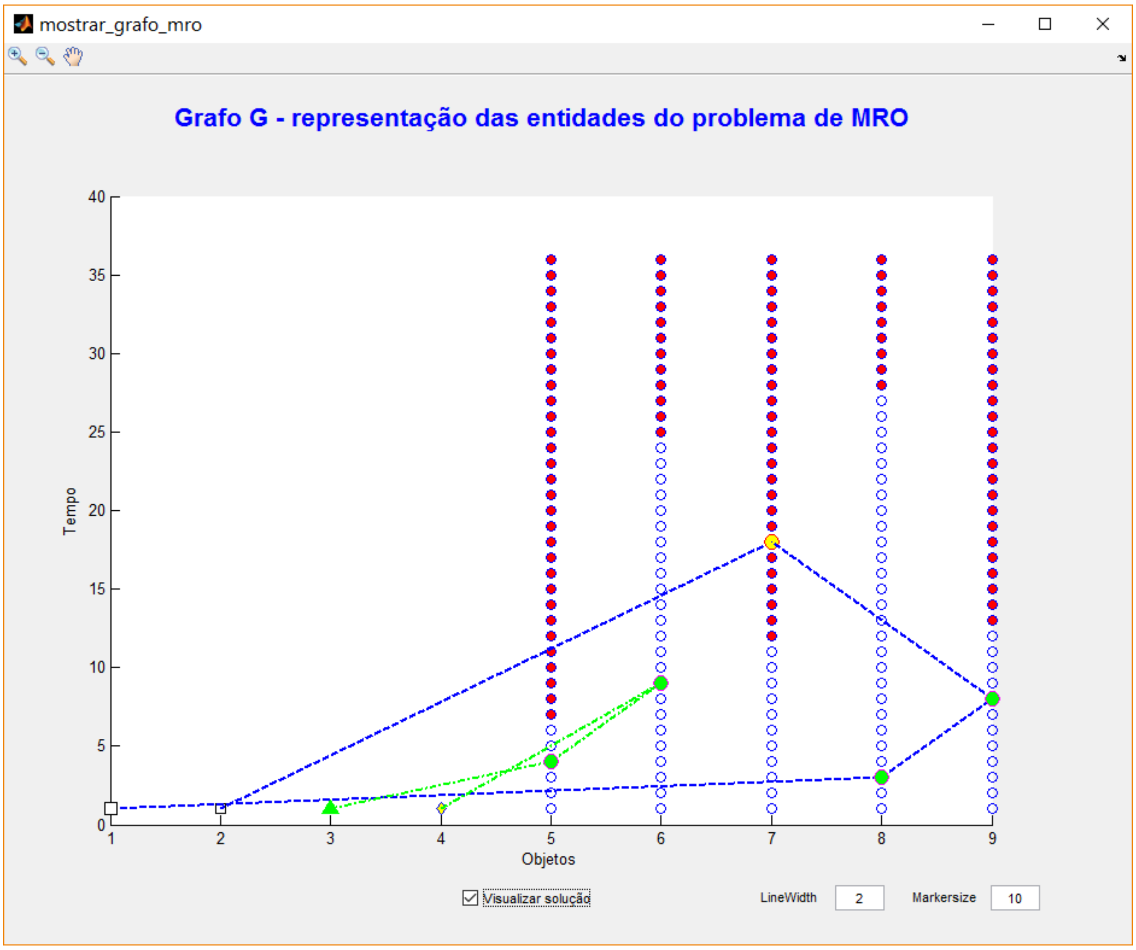

Figure 36. HC1d solution in layered graph

3.5.3 Efficacy of the solution

In the MMRO problem the merit of the rescue operation is measured by a linear function proportional to the time spent in the water and its values ranges from the instant the mission starts to the expected time of death for each survivor. For such a solution it is also possible to know the number of lives saved and also the number of survivors recovered without life and those that were not recovered. The effectiveness of a rescue operation can be measured by the number of lives, among those available, that are recovered alive. Nonetheless, more information can be collected and used to characterize the rescue operation. For example, data concerning the cost of using helicopters or other vehicles can be estimated and used to assess future budgets for the SAR System. Information regarding mission logistics such as fuel consumption, total flight hours, number of ships dispatched to the scene and the time they were committed to the rescue operations area are also variables that hold critical information to study strategical options for the organization of the SAR System. Information about the survivor's time in the water until recovery is also relevant to compare the efficiency of the vehicles involved.

The solution of the MMRO problem can be interpreted as a plan for the recovery of survivors prior to an accident. The purpose of such plan is to recover as many survivors alive as soon as possible. The plan does not take into account any constraints regarding the use of available resources. For example, the rescue plan is not built with a budget constraint because it wouldn't be ethically correct to do so. The general principle in constructing a rescue plan is to use all available resources guaranteeing the safety of crews and persons involved. So, why are we interested in estimating the cost of such rescue operation? The reason for collecting more information about the rescue operation which is not considered in the mathematical model has to do with the resources needed by the SAR System to cope with such an incident. Information regarding costs and vehicles availability is relevant to study the advantages of having more resources available.

The following interface resumes several statistical indicators that can be calculated from a MMRO solution:

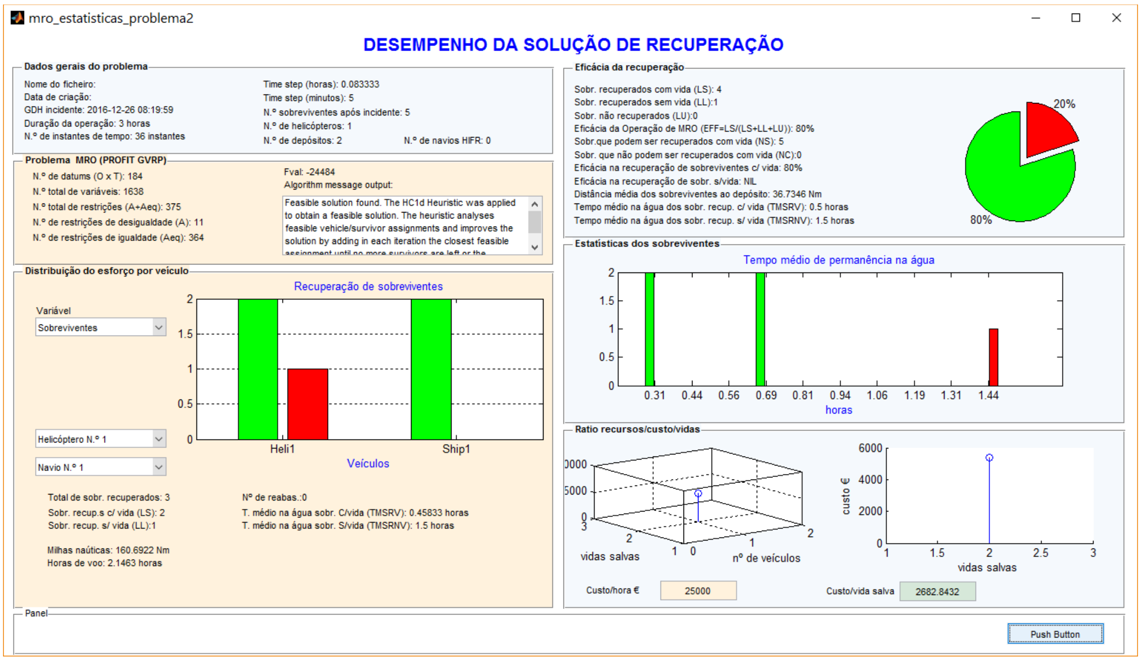

Figure 37. Performance of the MMRO solution

Several statistical indicators were made from a MMRO solution and they are associated to the efficacy of the MMRO solution but also to efficacy of the vehicles involved:

- MMRO solution performance indicators:
  - MMRO solution objective value.
  - Number of survivors recovered alive (lives saved – LS).
  - Number of survivors recovered without life (lives lost – LL).
  - Number of survivors not recovered (lives unaccounted for – LU).
  - Efficacy of rescue operation (EFF = (LS/(LS+LL+LU)).

    - Survivors that can be recovered alive (NS).
    - Survivors that cannot be recovered alive (NC).
    - Efficacy of recovering survivors that can be recovered alive (LS/NS).
    - Efficacy of recovering survivors that cannot be recovered alive (LL/NC).
    - Average time spent in water of survivors recovered alive.
    - Average time spent in water of recovered deceased survivors.
- Vehicles performance indicators:
    - Number of survivors recovered alive.
    - Number of survivors recovered without life.
    - Average time spent in water of survivors recovered alive.
    - Average time spent in water of recovered deceased survivors.
    - Total number of hours spent in operation.
    - Total number of miles travelled during operation.

The above indicators can be calculated from the solution of an MMRO instance for a specific maritime area of interest and these can be related to different types of large scale incidents. The pie chart is the main instrument to assess the efficacy of the rescue operation. For the MMRO example, the operation efficacy is 80% since only four out of five survivors were recovered alive. Since we assume to know the expected time of death of every survivor, it is possible to calculate *a priori* if a given survivor can be rescued by some vehicle while still being alive. This does not imply that it will be feasible to recover all of these survivors alive. Suppose there is only one vehicle and two survivors that have one hour to live each. The time to travel from the initial location of the vehicle to the location of each survivor is half an hour but both survivors distance each other one hour at the vehicle's speed. It is impossible to recover both survivors alive but both of them have the possibility to be recovered alive.

The presented indicators can differentiate solutions that have the same objective value or rescue operation efficacy. For example, two different plans may have the same efficacy but one of them may present a lower time in the water for the survivors recovered alive. It is important to phrase that a good rescue plan is one where the efficacy is high and the total time of its rescue activities is minimum, which is achieved by the proposed objective function. Meanwhile, information regarding the expected average of time spent in water estimated from several simulations using this model can be used as an argument to back political options regarding the acquirement of more resources, whether they are financial resources or equipment.

The underlying conclusion in this Section is that the MMRO model can be used to simulate scenarios where we admit incidents in a specific maritime area and wish to assess the response that is possible to achieve by the current facilities available to SAR authorities or other hypothetical facilities. These simulation scenarios can be evaluated in order to estimate the efficacy of the SAR System response to a variety of different settings. Different settings may be used, varying for example vehicle availability, in order to assess the impact of having one more or one less vehicle in the efficacy of the rescue operation.

3.5.4 Comparison of pilot method variations

Tailoring a pilot method for the MMRO problem requires the definition of several parameters and these must be consistent with the problem structure. For example, the proposed variants for the pilot method state that the assignments added to the master solution should account for each vehicle instead of being a subset of all feasible assignments that can be made from a partial solution. This tailoring choice is not mandatory but hinders the possibility of poor choices that may arise from consecutively assigning the same vehicle during the master solution construction. These parameters and how they are implemented have a huge impact on the method's performance. For this reason an interface was built to facilitate the comparison of different pilot methods variants for the same MMRO instance. The comparison is based on the objetive function value of the best known pilot solution in each iteration of the pilot method variant. Computational experimentation show that the pilot method does not improve the best known solution after a certain number of iterations (see Figure 38). These results were also observed by Fink and Voss [191] while solving the continuous flow-shop schedulling problem using several variants of a pilot method heuristic. Since the pilot method has a high complexity time, Fink and Voss studied the performance of the method while restricted to a certain *evaluation depth*. That is, the pilot method is performed until an incomplete solution with a given number of jobs is reached and this solution is completed by continuing with a conventional (myopic) cheapest insertion heuristic.

Figure 38 illustrates an MMRO problem with 20 survivors (PIW) and 4 vehicles (2 nearby ships and 2 helicopters) and two pilot variations:

- Pilot method variation A (3 levels with $m_1 = 6$, $m_2 = 6$ and $m_3 = 6$)
- Pilot method variation B (1 level with $m_1 = 4$)

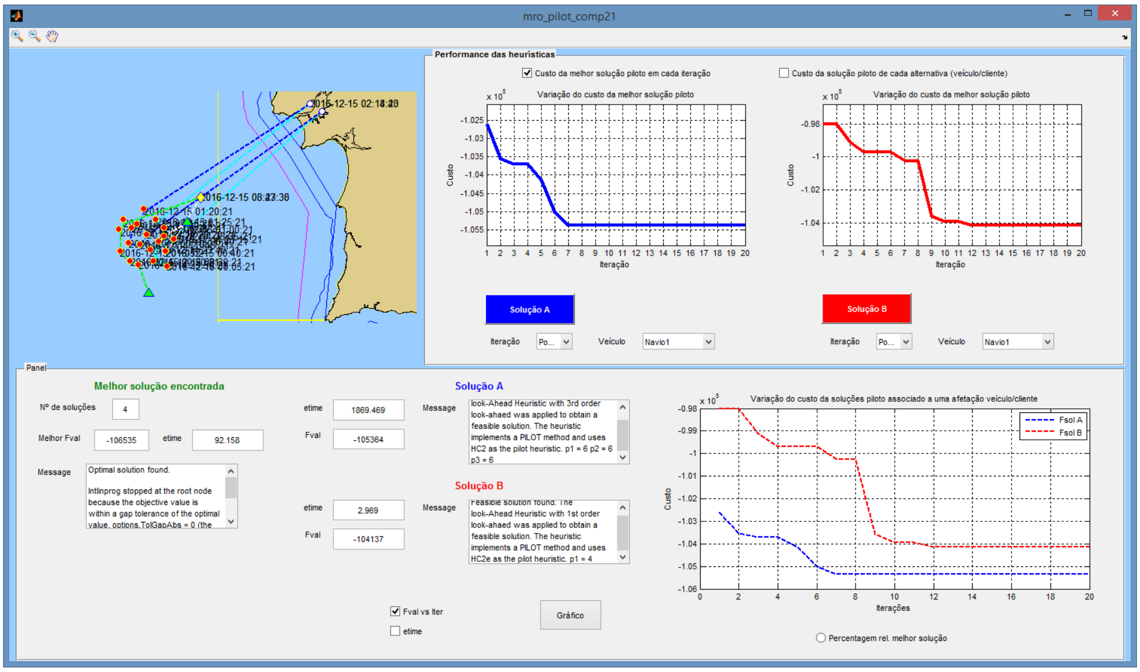

Figure 38. Interface to compare two pilot solutions

In the above problem, the optimal solution is known. Solution A (3 level pilot method) scored 98.9% and solution B scored 97.75% of the optimal value. For an improvement of 1.15% the elapsed time of variant A is six hundred times greater than the elapsed time of variant B.

To infer about the performance of the pilot method variations several indicators are collected during the algorithm execution:

- Total elapsed time. The time taken by the function that implements the pilot method to deliver a solution. This is measured using MATLAB *etime* function.
- Iteration elapsed time. The time taken at each iteration of the pilot method.
- Number of vehicle/survivor assignments evaluated in each level of the pilot method (may be lower than the maximum number parameter).
- Objective value of the best solution found at each iteration of the pilot method.

It is also relevant to calculate the ratios between the above indicators to compare solutions from different pilot variations. The variation of these ratios in each iteration can present some relevant insight about the performance of the methods for a given problem.

3.6 Computational experiments

The MMRO instances built to evaluate the performance of the heuristics are divided in two main groups: the first group is defined by the time-dependent objective function that simulates the survivor's remaining time alive and the second group is defined by a random profit for rescuing a survivor. One of the questions we are interested to answer is how the heuristics based on distance and expected time arrival fare compared with those that assume the remaining time to live is available for the elaboration of the rescue plan? This question only makes sense when the profit function decreases with time until the survivor's death. It is worth comparing the performance of the heuristics with "real" profits versus "random" ones. Which is also the same as comparing instances with a time dependent objective function and instances with random profits. Every instance of the "random" group of problems is obtained from its respective instance of the "real" group by randomly changing the profit values associated to each SAR object. The next subsections describe how the instances of the MMRO problem were built and the performance from the constructive heuristics and look ahead methods to solve those instances. These values are compared, when possible, with the optimal solution or linear relaxation.

3.6.1 Experimental data sets

All of the MMRO instances used to compared the heuristics (constructive and look-ahead) where created within the Portuguese continental Economic Exclusive Zone (EEZ) within Montijo's air base operational radius and also Lisbon international airport (Humberto Delgado airport) acting as a second possible depot for helicopters to end their tour. Weather information contained in GRIB files for the Portuguese EEZ location were used to take wind speed and direction for drift calculations. All the instances where built during December of 2016 and January of 2017, which is a period where weather conditions in the Atlantic are less agreeable for operating at sea. The centroid of the survivor's initial location to Montijo airbase ranges between 35 and 140 nautical miles. This last range is almost half way between Lisbon's coast line and the limit of the continental EEZ (quite far away for an immediate response). We assumed an average speed of 100 knots for helicopters and 10 knots for nearby ships dispatched to the scene. One of the shortfalls of the model is the impossibility to consider changes in speed while moving between time index nodes.

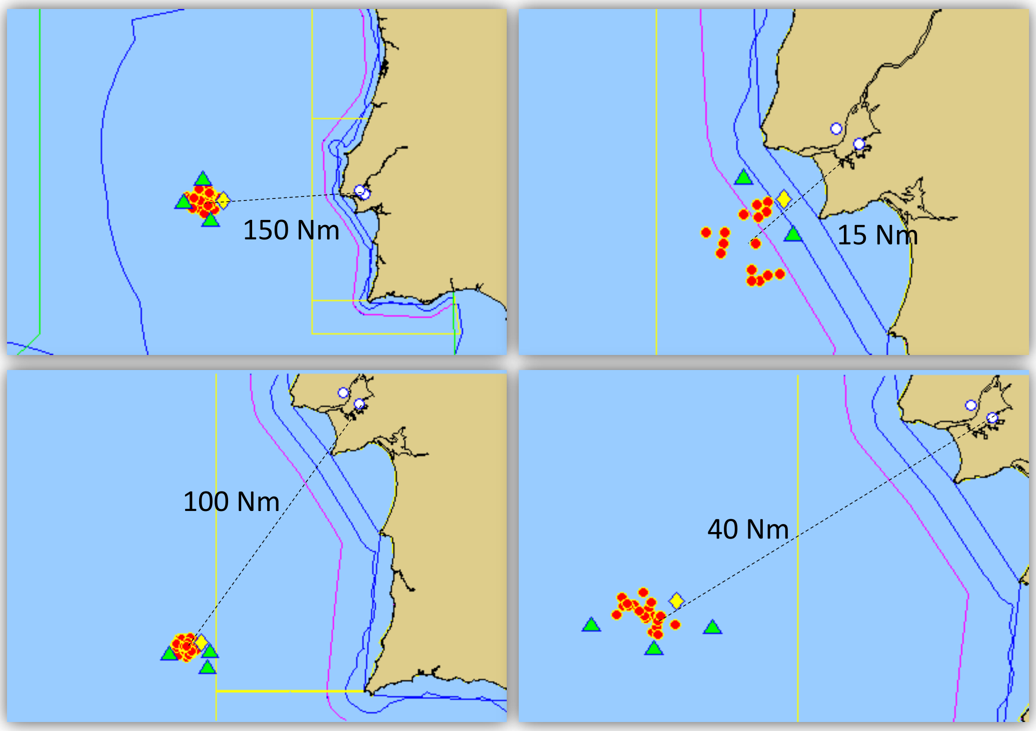

Figure 39. Survivor's and rescue unit's location for experimental MMRO instances

Up to a maximum of 5 vehicles are considered where the number of helicopters is never bigger than two. The reason for this choice has to do with the maximum number of helicopters that can be dispatched from Montijo's airbase in a short notice. More helicopters could be dispatched but would depart much later since they are not on call for such type of operation. The total number of survivors and depots correspond to the number of "clusters" of the problem and the number of time stamps corresponds to the cluster size in a GVRP statement of the problem. All survivors correspond to a person in the water on a vertical position. The vertical position of a person in the water is an indicator that the person is alive. Within each group of instances, these were organized into sets according to the number of vehicles and the number of survivors. The number of vehicles ranges from three to five with the following configuration:

- Three vehicles:
    - Two nearby ships and one helicopter.
- Four vehicles:
    - Three neraby ships and one helicopter.
- Five vehicles:
    - Three nearby ships and two helicopters.

For each set of instances, several statistics were calculated based on the average values of several variables that characterize the MRO instance. These variables include:

- Average survivor's drift distance. Average distance that a SAR object would drift during the mission duration measured in nautical miles (Nm).
- Average distance from survivors to their centroid. This variable measures the dispersion of the survivors.
- Average distance from helicopters to survivors at time 0.
- Average distance from nearby ships to survivors at time 0.
- Average survivors remaining time alive at time 0.
- Minimum remaining time alive at time 0.
- Maximum remaining time alive at time 0.

The expected time alive for each survivor is randomly generated with uniform distribution between the time 0 and the mission duration. The profit for rescuing a survivor is defined by a linear decay function of the remaining time alive. The profit for each assumes its maximum value at time zero (this value is set by the user) and the minimum value at the time instant where the survivor is deceased. The maximum and minimum values for the profit are parameters set by user when creating the MMRO instances.

The following tables describe several sets of instances of the MMRO problem where instances of the same set have the same number of vehicles and survivors.

Table 4. Instances description with 3 vehicles

Problem attribute	Instance sets				
Instance set category code	v3n15	v3n20	v3n30	v3n50	v3n80
Number of vehicles (helicopters + nearby ships)	3 (1+2)	3 (1+2)	3 (1+2)	3 (1+2)	3 (1+2)
Number of survivors	15	20	30	50	80
Mission duration (hours)	4	6	6	12	12
Time step (minutes)	3	3	3	5	5
Number of time stamps (layers in graph)	80	120	120	144	144
Number of nodes in layered graph	1205	2405	3605	7205	11525
Number of variables (binary integer programming problem)	54.660	145.380	435.960	1.080.150	2.757.840
Number of inequallity constraints (A)	31	41	62	101	161
Number of equality constraints (Aeq)	3606	7206	14408	21606	34566
Number of total constraints (A+Aeq)	3637	7247	14470	21707	34727
Average survivor's drift distance (Nm)	0,82	4,44	1,58	6,88	7,50
Average distance from survivors to their centroid (Nm)	7,76	4,92	4,25	5,24	2,66
Aaverage distance from helicopters to survivors at time 0 (Nm)	35,04	65,18	84,33	93,02	76,76
Average distance from nearby ships to survivors at time 0 (Nm)	14,42	19,30	12,50	18,74	12,45
Average survivor's remaining time to live (hours)	2,43	2,99	2,68	6,51	6,40
Minimum remaining time live (hours)	0,39	0,12	0,29	0,24	0,02
Maximum remaining time to live (hours)	3,88	5,92	5,97	11,89	11,98

Table 5. Instances description with 4 vehicles

Problem attribute	Instance sets				
Instance set category code	v4n15	v4n20	v4n30	v4n50	v4n80
Number of vehicles (helicopters + nearby ships)	4 (1+3)	4 (1+3)	4 (1+3)	4 (1+3)	4 (1+3)
Number of survivors	15	20	30	50	80
Mission duration (hours)	4	6	8	12	12
Time step (minutes)	3	3	5	5	5
Number of time stamps (layers in graph)	80	120	96	144	144
Number of nodes in layered graph	1205	2405	2886	7206	11526
Number of variables (binary integer programming problem)	73.680	195.440	345.330	1.437.950	3.673.520
Number of inequallity constraints (A)	32	42	61	101	161
Number of equality constraints (Aeq)	4808	9608	11528	28808	46088
Number of total constraints (A+Aeq)	4840	9650	11589	28909	46249
Average survivor's drift distance (Nm)	1,01	3,09	1,99	4,50	2,96
Average distance from survivors to their centroid (Nm)	5,53	5,22	5,47	5,08	4,63
Aaverage distance from helicopters to survivors at time 0 (Nm)	53,04	123,25	107,92	128,22	85,95
Average distance from nearby ships to survivors at time 0 (Nm)	13,34	30,85	15,42	18,67	18,49
Average survivor's remaining time to live (hours)	2,40	3,38	3,86	6,19	6,20
Minimum remaining time live (hours)	0,39	0,59	0,10	0,06	0,06
Maximum remaining time to live (hours)	3,88	5,82	7,98	11,65	11,65

Table 6. Instances description with 5 vehicles

Problem attribute	Instance sets				
Instance set category code	v5n15	v5n20	v5n30	v5n50	v5n80
Number of vehicles (helicopters + nearby ships)	5 (2+3)	5 (2+3)	5 (2+3)	5 (2+3)	5 (2+3)
Number of survivors	15	20	30	50	80
Mission duration (hours)	4	6	8	12	12
Time step (minutes)	3	3	5	5	5
Number of time stamps (layers in graph)	80	120	96	144	144
Number of nodes in layered graph	1206	2406	2886	7206	11526
Number of variables (binary integer programming problem)	91.545	243.160	433.800	1.802.800	4.600.480
Number of inequallity constraints (A)	32	42	62	102	162
Number of equality constraints (Aeq)	6010	12010	14410	36010	57610
Number of total constraints (A+Aeq)	6042	12052	14472	36112	57772
Average survivor's drift distance (Nm)	3,19	3,00	3,14	3,92	4,40
Average distance from survivors to their centroid (Nm)	3,91	4,02	4,37	4,91	4,63
Aaverage distance from helicopters to survivors at time 0 (Nm)	61,21	82,47	138,88	129,27	85,95
Average distance from nearby ships to survivors at time 0 (Nm)	15,18	27,51	17,40	16,13	18,49
Average survivor's remaining time to live (hours)	2,19	2,73	4,12	6,84	6,20
Minimum remaining time live (hours)	0,24	0,29	0,24	1,10	0,06
Maximum remaining time to live (hours)	3,83	5,97	7,88	11,95	11,65

A second group of sets were built from the initial sets where the objective function was changed in a way that the profit of rescuing a survivor is random. This can be done by randomly permuting the elements of the profit vector associated with each survivor. This was done using *permute* function from MATLAB. The first group of sets corresponds to the "real cost sets" and the second one is the random counterpart which we will refer as the "random cost sets". The difference between these group of sets remains only in the profit function of each instance. The remaining specifications are the same. The purpose of this second group of sets is to compare the performance of the heuristics, especially those that use information regarding distance and ETA as a criteria for choosing the vehicle/survivor assignment. In total, we have two groups of MMRO problem instances, organized in sets, where each group has fifteen sets in total.

3.6.2 Pilot method variants

Several look ahead variants were considered based upon the following features:

- pilot sub heuristic;
- assignment sequence length (or depth of the assignment tree);
- number of assignments evaluated in each iteration (complete vs partial);
- When not performing a complete evaluation of the feasible assignments but only a certain number (usually a small number due to computational time) then it is necessary to set a ranking criteria to choose those assignments. The criteria for ranking assignments to be evaluated are:
  - Distance between vehicle and survivor (minimum distance is preferred)
  - Expected time arrival (ETA) of vehicle to survivor location (minimum ETA is preferred);
  - Profit (profit gained from rescuing a survivor; maximum is preferred)

If only m assignments are evaluated and m is less than the maximum possible number of feasible assignments then a criteria must be specified to choose the m assignments. If the pilot method has l levels then in each level we can choose m_u assignments for $u = 1, \dots, l$.

As for the pilot sub heuristics we used all of the available constructive heuristics. Only two sequence lengths were used: a first level depth where only one assignment is added to the master solution ($l = 1$) and a two level depth where a sequence of two feasible assignments are added ($l = 2$). As for the number of assignments considered in each level, we considered a "full pilot method" where all feasible assignments in each level are considered as candidates to be added to the master solution and a "minimal pilot method" where only two assignments ($m_1 = 2, m_2 = 2$) in each level are considered. In this latter variation it is necessary to specify a criteria for choosing the m_1 and m_2 (whether it's a first level or a second level pilot method) assignments. In this "minimal variant", if we chose only one assignment (instead of two) the resulting pilot method would be similar to the constructive heuristics. The pilot method where in each level all feasible assignments are evaluated is called "complete pilot method". The pilot method where the number of feasible assignments to be evaluated is less than the maximum possible number is called “partial pilot method”. In this variant it is necessary to specify the criteria to choose the vehicle/survivor assignments to be evaluated. Three possible criteria are considered: distance criteria, ETA criteria and profit criteria. The last criteria assumes that the SAR system has full knowledge of the survival times of each survivor.

Table 7. Pilot method variants enumeration

variant (designation)	sub-heuristic	sequence length (number of levels)	number of assignments evaluated (in each level)	criteria for choosing assignments	variant (designation)	sub-heuristic	sequence length (number of levels)	number of assignments evaluated (in each level)	criteria for choosing assignments
HP1S1d	HC1d	1	all	-	HP1S1d_Pp	HC1d	1	m_1=2	profit
HP1S1e	HC1e	1	all	-	HP1S1e_Pp	HC1e	1	m_1=2	profit
HP1S1p	HC1p	1	all	-	HP1S1p_Pp	HC1p	1	m_1=2	profit
HP1S2d	HC2d	1	all	-	HP1S2d_Pp	HC2d	1	m_1=2	profit
HP1S2e	HC2e	1	all	-	HP1S2e_Pp	HC2e	1	m_1=2	profit
HP1S2p	HC2p	1	all	-	HP1S2p_Pp	HC2p	1	m_1=2	profit
HP2S1d	HC1d	2	all	-	HP2S1d_Pd	HC1d	2	m_1=2 m_2=2	distance
HP2S1e	HC1e	2	all	-	HP1S1e_Pd	HC1e	2	m_1=2 m_2=2	distance
HP2S1p	HC1p	2	all	-	HP1S1p_Pd	HC1p	2	m_1=2 m_2=2	distance
HP2S2d	HC2d	2	all	-	HP1S2d_Pd	HC2d	2	m_1=2 m_2=2	distance
HP2S2e	HC2e	2	all	-	HP1S2e_Pd	HC2e	2	m_1=2 m_2=2	distance
HP2S2p	HC2p	2	all	-	HP1S2p_Pd	HC2p	2	m_1=2 m_2=2	distance
HP1S1d_Pd	HC1d	1	m_1=2	distance	HP1S1d_Pe	HC1d	2	m_1=2 m_2=2	ETA
HP1S1e_Pd	HC1e	1	m_1=2	distance	HP1S1e_Pe	HC1e	2	m_1=2 m_2=2	ETA
HP1S1p_Pd	HC1p	1	m_1=2	distance	HP1S1p_Pe	HC1p	2	m_1=2 m_2=2	ETA
HP1S2d_Pd	HC2d	1	m_1=2	distance	HP1S2d_Pe	HC2d	2	m_1=2 m_2=2	ETA
HP1S2e_Pd	HC2e	1	m_1=2	distance	HP1S2e_Pe	HC2e	2	m_1=2 m_2=2	ETA
HP1S2p_Pd	HC2p	1	m_1=2	distance	HP1S2p_Pe	HC2p	2	m_1=2 m_2=2	ETA
HP1S1d_Pe	HC1d	1	m_1=2	ETA	HP1S1d_Pp	HC1d	2	m_1=2 m_2=2	profit
HP1S1e_Pe	HC1e	1	m_1=2	ETA	HP1S1e_Pp	HC1e	2	m_1=2 m_2=2	profit
HP1S1p_Pe	HC1p	1	m_1=2	ETA	HP1S1p_Pp	HC1p	2	m_1=2 m_2=2	profit
HP1S2d_Pe	HC2d	1	m_1=2	ETA	HP1S2d_Pp	HC2d	2	m_1=2 m_2=2	profit
HP1S2e_Pe	HC2e	1	m_1=2	ETA	HP1S2e_Pp	HC2e	2	m_1=2 m_2=2	profit
HP1S2p_Pe	HC2p	1	m_1=2	ETA	HP1S2p_Pp	HC2p	2	m_1=2 m_2=2	profit

3.6.3 Results

In this subsection, we provide computational results for the constructive heuristics and also the pilot method variations described in the previous subsection. The objective function of the heuristics and pilot method are compared with the optimal value whenever it is available. If the optimal value is not known then the linear relaxation optimal value is used. When no result is shown, it means that the linear relaxation value is not known. For the instances of the set v5n80, only the full pilot method was obtained which in turn was used to set the lower bound for the relative gap calculation in the constructive heuristic performance table. All heuristics (constructive and pilot method variants) were coded in MATLAB and run on a PC with a intel core i7 4820k CPU 3.70 Ghz and 32Gb RAM under Windows 10.

Real cost sets

The performance of the constructive heuristics and the pilot methods variants is measured by the maximum ($m\%$) and average gap ($a\%$) in percent of the optimal value (or a lower bound, when optimal value is unavailable). It is also presented the average elapsed time ($t\%$) measured with the MATLAB function *etime*. For each set of instances, the letters k, ns and np defines the number of vehicles, the number of survivors and the number of instances of each set, respectively. Table 8 shows the performance for the different variants of constructive heuristics on the fifteen sets of MMRO instances. The optimal value is only available for the sets with 15 and 20 survivors. The linear programming relaxation optimal value is used as a lower bound (MMRO is solved as a minimization problem) for the gap calculations in the sets v3n30, v3n50, v3n80, v4n30, v4n50, v4n80, v5n30 and v5n50. For the set v5n80 the linear relaxation was not possible to be calculated within the time limit of 5 days of CPU time (120 hours). For the set v5n80, the value of the full pilot method with one level was used as the lower bound for the maximum and average gap calculation.

Table 8. Stand-alone results for constructive heuristics

size			HC1d		HC1e		HC1p		HC2d		HC2e		HC2p	
k	ns	np	a%	m%	a%	m%	a%	m%	a%	m%	a%	m%	a%	m%
	15	30	19,64	44,57	20,20	45,53	14,62	26,40	15,82	40,43	17,16	47,34	28,44	46,64
	20	32	19,12	33,49	16,92	25,42	15,01	21,16	10,98	16,56	14,18	22,29	30,74	44,09
3	30	24	15,59	27,67	16,50	22,81	22,40	29,90	12,25	25,53	12,63	20,16	20,74	27,69
	50	10	16,82	20,99	16,39	25,27	25,35	30,46	12,89	17,13	13,58	20,01	24,27	28,54
	80	5	14,28	19,11	12,04	15,67	27,76	33,01	8,93	15,61	8,66	10,20	25,76	34,29
	15	30	20,59	44,06	22,51	52,36	14,69	25,08	12,02	24,42	12,86	34,97	16,68	29,31
	20	30	19,44	57,93	21,66	40,62	14,88	22,06	9,86	22,03	11,03	22,57	9,12	14,05
4	30	24	12,82	22,18	12,67	24,00	20,93	28,04	9,78	15,11	9,04	14,24	18,38	27,36
	50	10	17,02	22,53	15,92	19,03	24,21	27,19	12,44	19,87	11,57	19,70	21,53	26,08
	80	5	12,72	15,53	13,13	15,99	24,17	27,66	8,73	12,38	8,83	11,63	22,92	25,67
	15	30	13,32	26,87	12,86	29,33	16,26	28,46	9,68	23,73	9,38	21,58	13,78	27,76
	20	30	13,24	22,73	15,51	30,30	14,90	31,38	8,55	16,31	9,10	16,42	9,89	19,33
5	30	24	11,61	19,81	11,72	16,93	18,95	25,74	7,28	14,36	7,34	12,69	16,58	24,69
	50	10	13,79	20,77	13,82	22,37	21,26	26,80	10,41	16,00	10,74	17,10	19,68	22,62
	80	5	11,05	12,85	11,58	13,36	20,01	22,06	7,38	8,74	8,19	9,68	18,31	22,34
average			15,40	27,41	15,56	26,60	19,69	27,03	10,47	19,21	10,95	20,04	19,79	28,03

The greedy variants HC2d and HC2e provide the best performance considering the overall sets and, specially, with the larger instances (v3n80, v4n80 and v5n80).

Table 9 shows the performance of the full pilot method with one level variants based on the choice of the constructive heuristic acting as the pilot heuristic. The full pilot method outperforms the respective constructive heuristic used as pilot heuristic. For each set the full pilot method variants HC2d and HC2e managed to provide an average gap below 10%. Without a better lower bound for the instances in the set v5n80 it was not possible to estimate the average and maximum gap for the full pilot method variants with one level.

Table 9. Performance of full pilot methods variations with one level

size			HC1d		HC1e		HC1p		HC2d		HC2e		HC2p	
k	ns	np	a%	m%	a%	m%	a%	m%	a%	m%	a%	m%	a%	m%
3	15	30	5,43	13,24	6,01	13,56	4,61	14,36	2,16	8,77	2,22	6,04	2,32	10,81
	20	32	8,62	23,49	7,62	18,32	3,39	12,41	3,29	6,14	3,41	12,05	2,07	5,14
	30	24	9,28	13,65	9,62	15,52	11,83	19,13	5,06	13,42	4,77	9,70	8,26	13,66
	50	10	11,05	16,93	9,91	14,52	15,05	21,65	6,14	7,56	5,84	7,39	11,65	13,31
	80	5	15,85	19,13	15,31	17,67	21,63	29,80	9,77	14,07	9,67	14,01	17,88	22,41
4	15	30	4,56	12,47	5,71	12,64	2,47	11,01	1,85	5,84	1,61	4,62	1,66	6,51
	20	30	6,33	12,67	5,88	16,20	2,62	10,17	1,49	5,76	1,53	6,23	1,29	4,24
	30	24	5,55	11,25	5,84	10,90	9,11	18,23	2,94	6,79	2,62	6,18	5,87	9,65
	50	10	9,45	13,38	9,69	12,69	13,23	17,67	4,93	7,16	4,84	7,69	10,24	14,73
	80	5	15,29	19,03	14,67	16,54	20,25	22,57	8,14	9,11	7,76	8,55	15,35	17,61
5	15	30	4,76	12,77	3,60	9,53	3,08	14,29	0,92	3,51	1,01	2,69	1,34	3,37
	20	30	3,95	14,63	4,87	12,79	2,74	8,90	1,08	4,36	0,93	2,80	1,15	4,73
	30	24	3,95	8,34	4,13	6,65	8,39	14,94	2,17	4,33	1,93	3,24	4,98	11,11
	50	10	7,71	13,37	7,53	10,85	9,65	15,84	3,79	5,21	3,95	6,03	7,40	12,27
	80													
average			7,99	14,60	7,88	13,46	9,15	16,50	3,84	7,29	3,72	6,94	6,53	10,68

Table 10 and Table 11 show the elapsed time for the constructive heuristics and full pilot method variants, respectively.

Table 10. Time performance for stand-alone constructive heuristics variations

size			HC1d	HC1e	HC1p	HC2d	HC2e	HC2p
k	ns	np	t%	t%	t%	t%	t%	t%
3	15	30	0.0 secs	0.0 secs	0.0 secs	0.0 secs	0.0 secs	0.0 secs
	20	32	0.0 secs	0.0 secs	0.0 secs	0.1 secs	0.0 secs	0.0 secs
	30	24	0.0 secs	0.0 secs	0.0 secs	0.1 secs	0.1 secs	0.1 secs
	50	10	0.1 secs	0.1 secs	0.1 secs	0.3 secs	0.2 secs	0.2 secs
	80	5	0.2 secs	0.3 secs	0.2 secs	0.8 secs	0.4 secs	0.4 secs
4	15	30	0.0 secs	0.0 secs	0.0 secs	0.0 secs	0.0 secs	0.0 secs
	20	30	0.0 secs	0.0 secs	0.0 secs	0.1 secs	0.1 secs	0.1 secs
	30	24	0.0 secs	0.0 secs	0.0 secs	0.2 secs	0.1 secs	0.1 secs
	50	10	0.1 secs	0.1 secs	0.1 secs	0.4 secs	0.3 secs	0.3 secs
	80	5	0.2 secs	0.4 secs	0.2 secs	1.3 secs	0.7 secs	0.7 secs
5	15	30	0.0 secs	0.0 secs	0.0 secs	0.0 secs	0.0 secs	0.0 secs
	20	30	0.0 secs	0.0 secs	0.0 secs	0.1 secs	0.1 secs	0.1 secs
	30	24	0.0 secs	0.0 secs	0.0 secs	0.1 secs	0.1 secs	0.1 secs
	50	10	0.1 secs	0.1 secs	0.1 secs	0.5 secs	0.3 secs	0.3 secs
	80	5	0.2 secs	0.4 secs	0.2 secs	1.6 secs	0.8 secs	0.8 secs

The full pilot method with the greedy constructive heuristic acting as the pilot heuristic presents higher running times when compared with the simple sequential constructive heuristic. It is worth noting that the full pilot method with HC1e as pilot heuristic presents higher running times when compared with the variants that use HC1d and HC1p. The higher running time is due to the calculations necessary to find the time required by each vehicle to reach a certain SAR object. This calculation is not necessary when using distance or profit.

Table 11. Time performance for full pilot method variations with one level

size			HC1d	HC1e	HC1p	HC2d	HC2e	HC2p
k	ns	np	t%	t%	t%	t%	t%	t%
3	15	30	3.5 secs	3.8 secs	3.6 secs	5.2 secs	5.0 secs	2.7 secs
	20	32	8.3 secs	9.7 secs	8.9 secs	13.8 secs	13.6 secs	7.0 secs
	30	24	21.8 secs	29.3 secs	22.6 secs	1 min, 3.2 secs	1 min, 2.2 secs	44.3 secs
	50	10	2 mins, 28.8 secs	4 mins, 3.2 secs	2 mins, 36.5 secs	6 mins, 30.9 secs	6 mins, 35.8 secs	6 mins, 18.2 secs
	80	5	19 mins, 22.8 secs	34 mins, 25.2 secs	19 mins, 3.2 secs	49 mins, 16.7 secs	50 mins, 49.9 secs	50 mins, 11.2 secs
4	15	30	1.2 secs	1.8 secs	1.3 secs	5.4 secs	5.2 secs	5.6 secs
	20	30	9.1 secs	11.1 secs	9.4 secs	22.5 secs	22.4 secs	16.3 secs
	30	24	17.0 secs	25.8 secs	17.5 secs	1 min, 18.8 secs	1 min, 17.4 secs	1 min, 22.0 secs
	50	10	3 mins, 14.0 secs	5 mins, 24.8 secs	3 mins, 33.0 secs	10 mins, 25.0 secs	10 mins, 28.6 secs	10 mins, 57.7 secs
	80	5	23 mins, 42.3 secs	42 mins, 59.1 secs	27 mins, 18.9 secs	1 hour, 10 mins, 32.9 secs	1 hour, 12 mins, 19.8 secs	1 hour, 12 mins, 22.2 secs
5	15	30	2.9 secs	3.4 secs	3.0 secs	6.6 secs	6.4 secs	7.6 secs
	20	30	9.9 secs	12.1 secs	10.1 secs	26.7 secs	26.2 secs	26.6 secs
	30	24	30.9 secs	41.9 secs	31.6 secs	1 min, 53.1 secs	1 min, 52.4 secs	1 min, 39.5 secs
	50	10	3 mins, 52.1 secs	6 mins, 37.3 secs	4 mins, 7.7 secs	14 mins, 25.5 secs	14 mins, 36.6 secs	14 mins, 21.3 secs
	80	5	30 mins, 12.0 secs	54 mins, 34.5 secs	33 mins, 16.7 secs	1 hour, 44 mins, 21.7 secs	1 hour, 48 mins, 23.0 secs	1 hour, 45 mins, 13.6 secs

Table 12 shows the performance of the full pilot method with two levels. The results show an improved performance when compared with the full pilot mtehod with one level.

Table 12. Performance of full pilot methods variations with two levels

size			HC1d		HC1e		HC1p		HC2d		HC2e		HC2p	
k	ns	np	a%	m%	a%	m%	a%	m%	a%	m%	a%	m%	a%	m%
3	15	30	2,08	6,79	2,13	5,47	1,69	7,97	1,04	5,17	1,06	4,12	1,13	6,39
	20	32	3,26	7,46	2,95	6,75	1,72	11,61	1,48	5,13	1,33	3,13	1,02	3,25
	30	24	5,00	10,54	6,00	10,55	6,16	9,19	3,21	6,20	3,36	7,05	5,41	13,46
	50	2	10,41	13,25	8,10	9,44	10,91	11,13	4,28	4,90	4,49	5,95	9,16	10,77
	80													
4	15	30	2,71	10,02	2,40	5,80	1,18	3,89	0,54	2,93	0,44	1,77	0,95	4,68
	20	30	3,15	7,89	2,52	11,22	1,38	5,12	0,60	3,48	0,56	3,19	0,73	3,46
	30	24	3,44	6,37	2,91	6,39	4,93	11,26	2,12	4,26	2,00	5,44	4,51	11,84
	50	2	5,80	7,95	4,73	4,88	10,35	12,81	2,74	3,01	3,17	3,58	6,20	6,52
	80													
5	15	30	2,22	9,82	1,78	7,96	1,83	5,77	0,48	3,16	0,37	1,33	0,78	3,40
	20	30	2,30	6,13	1,93	7,98	1,41	6,95	0,37	1,64	0,30	1,44	0,55	2,99
	30													
	50													
	80													
average			4,04	8,62	3,55	7,64	4,15	8,57	1,68	3,99	1,71	3,70	3,04	6,68

Table 13 shows the running times for the full pilot method with two levels. It is clear the high running times associated with larger instances of the MMRO problem. For example, the instances in the set v3n30 are solved between 10 and 43 minutes among all variants and these values increase almost 20 times (solved between 4 and 11 hours) when the number of SAR objects increase from 30 to 50. With four vehicles, the running times almost increase exponentially with the number of SAR objects.

Table 13. Time performance for full pilot method variations with two levels

size			HC1d	HC1e	HC1p	HC2d	HC2e	HC2p
k	ns	np	t%	t%	t%	t%	t%	t%
	15	30	25.0 secs	29.8 secs	26.5 secs	1 min, 25.2 secs	1 min, 24.2 secs	53.4 secs
	20	32	1 min, 32.9 secs	2 mins, 8.1 secs	1 min, 45.0 secs	5 mins, 7.8 secs	5 mins, 16.0 secs	3 mins, 53.7 secs
3	30	24	11 mins, 22.0 secs	16 mins, 50.3 secs	12 mins, 14.4 secs	38 mins, 28.8 secs	38 mins, 36.9 secs	43 mins, 30.8 secs
	50	2	4 hours, 6 mins, 58.0 secs	6 hours, 42 mins, 51.2 secs	4 hours, 14 mins, 34.4 secs	10 hours, 43 mins, 22.2 secs	11 hours, 13 mins, 21.5 secs	10 hours, 58 mins, 19.4 secs
	80							
	15	30	38.8 secs	47.6 secs	40.8 secs	2 mins, 30.1 secs	2 mins, 32.8 secs	2 mins, 1.0 secs
	20	30	2 mins, 45.9 secs	3 mins, 44.2 secs	2 mins, 55.4 secs	11 mins, 2.1 secs	11 mins, 11.9 secs	8 mins, 11.2 secs
4	30	24	21 mins, 52.6 secs	31 mins, 59.6 secs	22 mins, 48.0 secs	1 hour, 26 mins, 9.0 secs	1 hour, 27 mins, 32.9 secs	1 hour, 38 mins, 49.0 secs
	50	10						
	80	5						
	15	30	1 min, 4.3 secs	1 min, 19.4 secs	1 min, 8.6 secs	5 mins, 1.7 secs	4 mins, 56.3 secs	3 mins, 42.9 secs
	20	30	4 mins, 20.1 secs	5 mins, 55.2 secs	4 mins, 33.8 secs	20 mins, 41.4 secs	20 mins, 47.7 secs	23 mins, 59.4 secs
5	30							
	50							
	80							

For larger instances it is prohibitive to apply the full pilot method with two levels due to the high runnning times involved. For this reason the full pilot method with two levels was not applied to the sets v3n80, v4n50, v4n80, v5n30, v5n50 and v5n80.

Table 14, Table 15 and Table 16 show the performance of the partial pilot method with distance, ETA and profit criteria, respectively.

Table 14. Performance of partial pilot method variations with one level and distance criteria

size			HC1d		HC1e		HC1p		HC2d		HC2e		HC2p	
k	ns	np	a%	m%	a%	m%	a%	m%	a%	m%	a%	m%	a%	m%
	15	30	9,41	19,88	9,17	21,11	7,67	25,47	5,17	20,16	4,53	15,93	4,19	11,98
	20	32	9,59	19,39	9,39	20,70	8,75	36,24	5,23	10,34	4,63	12,54	4,55	15,30
3	30	24	9,90	15,77	9,58	16,86	10,78	17,25	7,47	12,11	6,80	13,03	8,02	16,22
	50	10	10,60	13,05	10,66	18,39	13,53	18,12	8,61	11,92	8,86	11,33	10,72	13,15
	80	5	13,32	16,35	15,09	20,19	19,81	22,32	12,97	20,15	11,56	14,55	16,03	19,66
	15	30	7,17	22,49	7,21	17,71	6,69	27,32	3,42	9,05	3,66	10,38	2,68	7,32
	20	30	9,87	26,93	9,10	18,91	6,89	16,85	3,04	7,52	3,02	6,70	3,10	8,75
4	30	24	6,87	15,91	7,37	13,92	8,16	13,36	5,49	11,32	4,47	8,63	5,40	12,57
	50	10	9,17	10,63	9,57	11,71	11,47	14,28	7,83	9,82	6,65	9,52	8,57	10,42
	80	5	12,74	13,34	12,73	14,76	15,54	16,21	12,88	14,87	9,88	11,84	13,61	15,37
	15	30	6,77	15,49	7,78	25,14	6,72	12,91	3,35	13,54	2,76	13,54	3,64	11,57
	20	30	7,62	27,63	7,07	14,57	6,75	16,04	3,00	8,24	2,50	4,92	3,01	8,75
5	30	24	5,79	11,10	6,35	12,72	7,63	14,21	3,75	6,50	3,18	5,25	4,87	9,86
	50	10	8,15	12,75	8,88	15,13	9,08	11,78	6,14	10,19	5,82	8,85	7,43	10,80
	80													
average			9,07	17,19	9,28	17,27	9,96	18,74	6,31	11,84	5,59	10,50	6,84	12,27

The variants with the ETA criteria (Table 15) present a similar performance among all variants when compared with distance (Table 14) variants. The variants with the profit criteria (Table 16) present worst performances when compared with ETA and distance variants. The variants with the pilot heuristic HC2e and HC2d present better performances when compared to variants with different pilot heuristics.

Table 15. Performance of partial pilot method variations with one level and ETA criteria

size			HC1d		HC1e		HC1p		HC2d		HC2e		HC2p	
k	ns	np	a%	m%	a%	m%	a%	m%	a%	m%	a%	m%	a%	m%
	15	30	10,72	25,40	8,66	24,32	7,28	31,18	4,98	15,21	4,87	16,19	2,82	9,13
	20	32	9,98	16,06	9,87	19,48	6,46	14,56	5,36	19,36	5,02	19,01	3,96	8,61
3	30	24	9,08	14,96	10,15	17,02	11,71	23,91	7,41	14,99	6,49	12,42	8,62	23,17
	50	10	10,61	13,68	9,70	13,56	14,58	22,07	7,29	9,82	7,72	9,30	12,56	16,93
	80	5	13,90	17,68	14,82	17,84	18,74	26,16	11,92	13,87	12,03	15,22	15,83	20,85
	15	30	9,21	20,59	8,21	22,99	6,42	15,24	3,19	15,18	3,30	14,89	2,86	10,74
	20	30	10,98	20,89	10,12	18,78	7,53	16,29	3,32	8,80	3,26	7,42	3,08	7,36
4	30	24	6,39	10,67	6,95	12,85	8,54	14,80	5,51	10,40	4,23	10,12	5,66	11,03
	50	10	10,21	13,49	9,30	11,62	10,44	12,07	6,70	8,77	7,12	8,86	9,42	13,14
	80	5	13,52	17,84	11,73	13,52	16,27	19,60	12,26	14,83	10,66	11,67	11,84	13,64
	15	30	8,26	18,81	6,94	15,69	6,58	12,92	3,20	13,54	2,53	10,77	3,28	9,40
	20	30	7,45	25,87	7,34	17,19	6,20	12,93	3,37	8,80	2,84	7,04	3,01	7,36
5	30	24	5,91	10,46	5,96	9,50	7,59	13,60	4,04	8,23	3,53	7,21	5,36	8,98
	50	10	8,27	12,15	7,99	14,17	8,93	10,59	6,09	9,02	5,55	7,74	6,75	7,64
	80													
average			9,61	17,04	9,12	16,32	9,81	17,57	6,05	12,20	5,65	11,28	6,79	12,00

Table 16. Performance of partial pilot method variations with one level and profit criteria

size			HC1d		HC1e		HC1p		HC2d		HC2e		HC2p	
k	ns	np	a%	m%	a%	m%	a%	m%	a%	m%	a%	m%	a%	m%
	15	30	9,13	15,27	10,21	20,33	8,73	17,42	7,55	15,43	7,96	17,35	6,75	13,39
	20	32	11,26	20,61	10,01	20,31	7,71	14,86	9,38	17,49	9,98	22,19	6,87	17,17
3	30	24	17,45	23,16	17,58	24,86	15,86	20,15	16,02	26,46	16,47	24,41	15,34	22,69
	50	10	21,55	26,08	22,11	28,40	20,73	26,13	22,52	30,57	21,31	27,85	21,04	25,98
	80	5	27,96	31,74	29,28	32,46	27,65	33,60	31,78	35,16	29,46	34,80	28,47	31,95
	15	30	10,46	20,96	9,90	19,65	9,29	15,92	6,89	15,62	6,62	14,14	6,62	12,57
	20	30	8,58	12,95	8,22	17,23	8,48	13,85	6,65	14,15	6,64	10,97	5,77	9,67
4	30	24	13,67	22,74	13,72	21,75	13,76	20,90	12,41	18,88	11,63	16,32	12,76	21,92
	50	10	17,29	25,85	17,79	22,54	19,23	24,32	18,69	25,01	17,78	22,50	17,38	23,78
	80	5	26,46	31,35	23,15	24,33	24,47	27,67	27,04	31,95	25,84	28,03	24,64	28,72
	15	30	8,78	19,96	8,42	13,47	8,06	16,03	5,86	13,59	5,23	12,98	6,15	13,17
	20	30	8,01	24,17	8,07	16,78	7,64	16,13	5,65	13,88	5,46	14,40	6,03	12,82
5	30	24	10,92	15,63	10,87	16,65	11,61	18,03	9,44	19,46	8,83	15,06	11,89	18,22
	50	10	15,59	19,48	15,69	18,54	16,23	19,53	16,07	24,49	14,94	21,83	15,35	20,77
	80													
average			14,79	22,14	14,64	21,24	14,25	20,32	14,00	21,58	13,44	20,20	13,22	19,49

Table 17, Table 18 and Table 19 show the performance of the partial pilot method with two levels with distance, ETA and profit criteria, respectively. The performance of the ETA and distance variants is similar to the one found among the variants of the partial pilot method with one level and ETA and distance criteria. Overall, the partial pilot method with two levels outperforms the respective variant in the partial pilot method with one level.

Table 17. Performance of partial pilot method with two levels and distance criteria

size			HC1d		HC1e		HC1p		HC2d		HC2e		HC2p	
k	ns	np	a%	m%	a%	m%	a%	m%	a%	m%	a%	m%	a%	m%
	15	30	6,25	17,76	6,08	15,62	5,11	14,75	3,24	11,11	3,67	15,51	2,81	10,33
	20	32	7,77	16,21	7,55	13,90	6,34	17,64	4,62	19,11	4,41	12,61	5,03	19,93
3	30	24	7,91	14,37	8,19	14,03	7,59	14,90	6,29	11,04	5,77	12,15	6,84	11,17
	50	10	9,05	11,72	9,00	12,70	11,56	16,75	7,75	10,36	7,77	11,17	11,51	17,27
	80	5	14,43	19,11	14,02	17,10	14,74	17,42	14,70	20,21	11,26	15,25	13,45	18,62
	15	30	5,83	22,49	5,61	15,59	4,56	16,09	2,68	8,51	2,55	8,51	2,25	5,18
	20	30	7,75	15,79	7,67	16,83	5,85	13,86	2,89	10,12	3,10	11,44	1,95	5,17
4	30	24	6,15	11,13	4,70	8,38	5,56	11,74	4,61	10,80	3,93	7,33	4,11	8,77
	50	10	7,59	9,59	8,60	12,01	9,59	12,90	7,00	8,68	6,69	9,43	7,52	10,27
	80	5	11,66	14,62	11,54	12,76	12,77	15,70	10,98	13,63	9,36	11,28	11,97	16,71
	15	30	5,51	16,66	4,34	14,68	4,58	9,73	1,84	5,13	2,24	13,54	2,38	10,86
	20	30	5,24	10,22	5,69	15,75	5,03	9,67	2,39	10,12	2,19	11,44	1,65	3,98
5	30	24	4,68	8,25	4,58	8,31	5,76	9,90	3,11	5,94	2,87	5,49	3,44	5,50
	50	10	6,45	10,07	6,82	12,16	7,29	10,53	6,23	9,60	5,13	7,02	6,89	9,88
	80													
average			7,59	14,14	7,46	13,56	7,60	13,69	5,59	11,03	5,07	10,87	5,84	10,97

Table 18. Performance of partial pilot method with two levels and ETA criteria

size			HC1d		HC1e		HC1p		HC2d		HC2e		HC2p	
k	ns	np	a%	m%	a%	m%	a%	m%	a%	m%	a%	m%	a%	m%
	15	30	6,57	17,26	6,58	20,57	4,91	14,75	3,37	10,11	3,10	9,14	2,67	7,67
	20	32	7,85	13,43	7,86	20,85	6,29	15,08	4,92	9,54	3,63	8,41	3,09	9,21
3	30	24	7,91	12,66	7,32	13,44	8,17	12,72	6,20	11,68	5,35	9,49	6,79	14,25
	50	10	9,63	12,61	7,94	12,49	10,45	17,36	7,42	11,17	6,59	9,96	8,54	11,79
	80	5	13,63	20,14	12,68	13,97	13,69	17,54	13,23	16,15	11,15	14,39	12,14	15,07
	15	30	5,85	14,04	5,70	15,55	5,54	15,56	2,05	5,75	2,37	7,79	2,37	6,72
	20	30	7,32	16,38	7,22	15,52	6,00	13,27	2,93	9,29	2,99	10,88	2,50	4,67
4	30	24	5,45	10,65	5,24	11,51	6,23	9,54	4,35	8,20	3,45	6,96	4,46	8,93
	50	10	7,76	10,80	7,22	10,33	9,81	12,79	6,96	11,85	5,94	7,50	7,32	8,91
	80	5	10,86	12,70	11,16	12,97	12,48	14,23	10,69	12,85	9,21	9,96	11,30	13,60
	15	30	5,20	12,84	4,59	11,33	4,42	14,60	1,64	4,11	2,06	10,35	2,37	8,80
	20	30	6,02	16,37	6,05	33,97	4,97	10,19	2,46	9,29	2,21	6,46	2,03	5,75
5	30	24	4,37	7,58	4,64	7,87	5,56	8,56	3,08	6,15	3,02	5,75	3,58	5,61
	50	10	7,28	9,89	6,44	9,84	7,00	8,93	5,24	7,26	4,93	7,17	5,61	6,97
	80													
average			7,55	13,38	7,19	15,01	7,54	13,22	5,32	9,53	4,71	8,87	5,34	9,14

Table 19. Performance of partial pilot method variations with two levels and profit criteria

size			HC1d		HC1e		HC1p		HC2d		HC2e		HC2p	
k	ns	np	a%	m%	a%	m%	a%	m%	a%	m%	a%	m%	a%	m%
	15	30	7,96	17,65	7,58	15,36	7,29	16,79	6,52	13,90	6,57	13,58	5,26	12,47
	20	32	7,79	14,28	7,87	15,87	7,03	13,59	7,62	14,15	7,02	12,75	5,84	13,35
3	30	24	15,72	23,54	15,91	22,96	13,91	18,95	14,54	21,16	14,10	20,06	13,94	18,14
	50	10	20,67	24,93	20,94	27,11	19,04	22,59	21,52	31,74	21,00	26,97	19,98	24,92
	80	5	29,33	31,58	29,52	34,23	28,25	32,59	28,49	32,28	29,48	33,07	27,68	33,16
	15	30	7,64	14,83	7,96	17,19	7,11	12,58	5,45	13,42	5,83	12,81	4,93	9,29
	20	30	6,95	11,66	7,54	13,36	6,46	13,16	5,19	8,87	5,67	11,62	4,87	8,93
4	30	24	12,06	19,08	11,50	18,14	11,58	18,56	11,28	16,34	10,84	17,32	11,65	18,53
	50	10	17,18	21,10	16,72	21,45	17,30	22,56	17,45	26,26	16,28	23,81	16,49	22,85
	80	5	25,48	29,86	23,37	24,05	21,97	24,12	26,01	30,44	26,06	29,59	22,70	24,86
	15	30	6,78	16,13	6,77	12,53	6,87	11,96	4,83	9,62	3,90	7,71	4,60	10,00
	20	30	6,27	12,33	6,62	13,99	6,19	10,93	4,95	10,02	4,53	12,73	4,72	7,88
5	30	24	9,39	13,82	9,06	12,95	11,21	19,20	8,58	14,52	8,29	15,75	9,87	15,10
	50	10	14,64	20,22	14,17	17,12	14,71	20,06	14,84	18,71	13,82	16,63	13,85	20,08
	80													
average			13,42	19,36	13,25	19,02	12,78	18,40	12,66	18,67	12,39	18,17	11,88	17,11

The pilot method variants with the profit criteria are outperformed by the variants with the ETA and distance criteria in the methods with one and two levels. Among all partial pilot method variants, the partial pilot method with two levels using HC2e as pilot heuristic and ETA as the decision criteria (Table 18) to choose the vehicle/survivors assignments to evaluate in each step was the one who presented the overall best performance among all sets. The time performance for the partial pilot method with two levels variants are presented in Appendix A - Time performance for partial pilot method variants.

Table 20 shows the time performance for the LP relaxation and optimal solution obtained by MATLAB *intlinprog* algorithm. Even the LP relaxation takes a prohibitive expensive time for sets with more than 50 SAR objects.

Table 20. Time performance of Branch-and-bound algorithm within MATLAB intlinprog function

size			Linear relaxation time		Optimal solution time	
k	ns	np	a%	m%	a%	m%
	15	30	3.3 secs	6.1 secs	42.7 secs	8 mins, 43.9 secs
	20	32	37.9 secs	2 mins, 44.2 secs	52.8 secs	3 mins, 50.8 secs
3	30	24	9 mins, 9.5 secs	24 mins, 23.7 secs		
	50	10	10 hours, 48 mins, 23.6 secs	13 hours, 39 mins, 28.9 secs		
	80		59 hours, 19 mins, 25.1 secs	75 hours, 5 mins, 27.9 secs		
	15	30	7.2 secs	12.3 secs	14.5 secs	42.9 secs
	20	30	2 mins, 21.4 secs	9 mins, 25.3 secs	10 mins, 50.2 secs	34 mins, 33.2 secs
4	30	24	10 mins, 39.3 secs	24 mins, 28.2 secs		
	50	10	11 hours, 9 mins, 25.3 secs	24 hours, 57 mins, 20.9 secs		
	80		96 hours, 34 mins, 43.1 secs	119 hours, 36 mins, 46.6 secs		
	15	30	7.2 secs	11.5 secs	1 min, 3.3 secs	10 mins, 23.6 secs
	20	30	2 mins, 31.2 secs	5 mins, 12.9 secs	19 mins, 34.9 secs	50 mins, 0.3 secs
5	30	24	14 mins, 37.3 secs	23 mins, 5.1 secs		
	50	10	13 hours, 3 mins, 11.5 secs	24 hours, 36 mins, 16.0 secs		
	80					

Figure 40 resumes the average performance for the constructive heuristics and full pilot method with one and two levels among the sets for which it was possible to apply all of these heuristics. The full pilot method with two levels is not an option for larger instances due to its prohibitive running time.

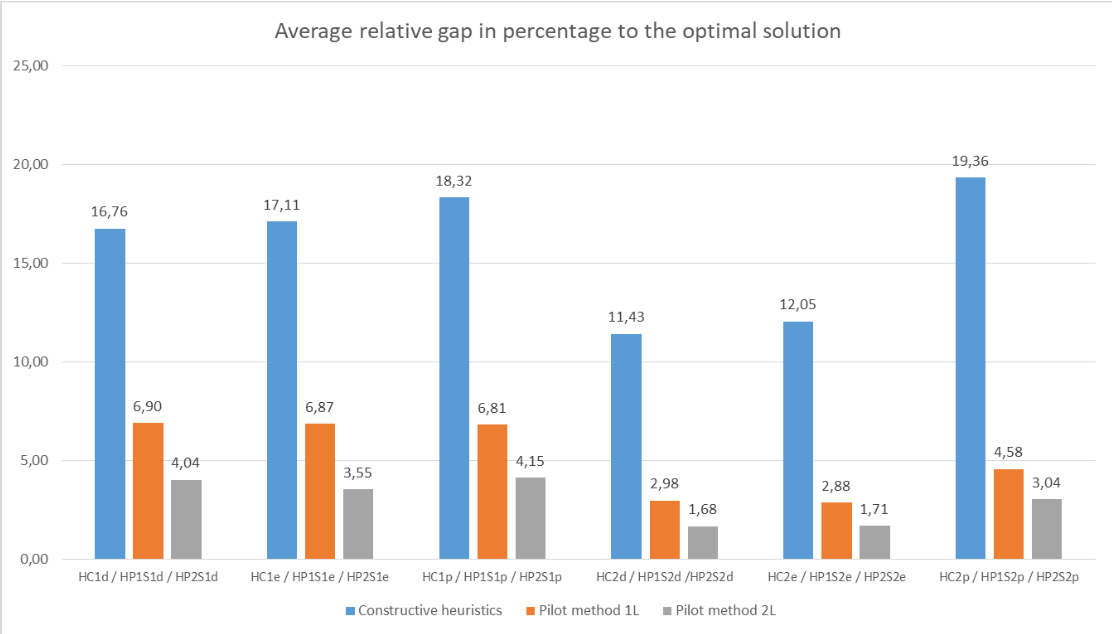

Figure 40. Performance for constructive heuristics and full pilot methods variants with one and two levels

Figure 41 and Figure 42 compare the performance of the variants that have different criteria for the partial pilot method with one level and two levels, respectively.

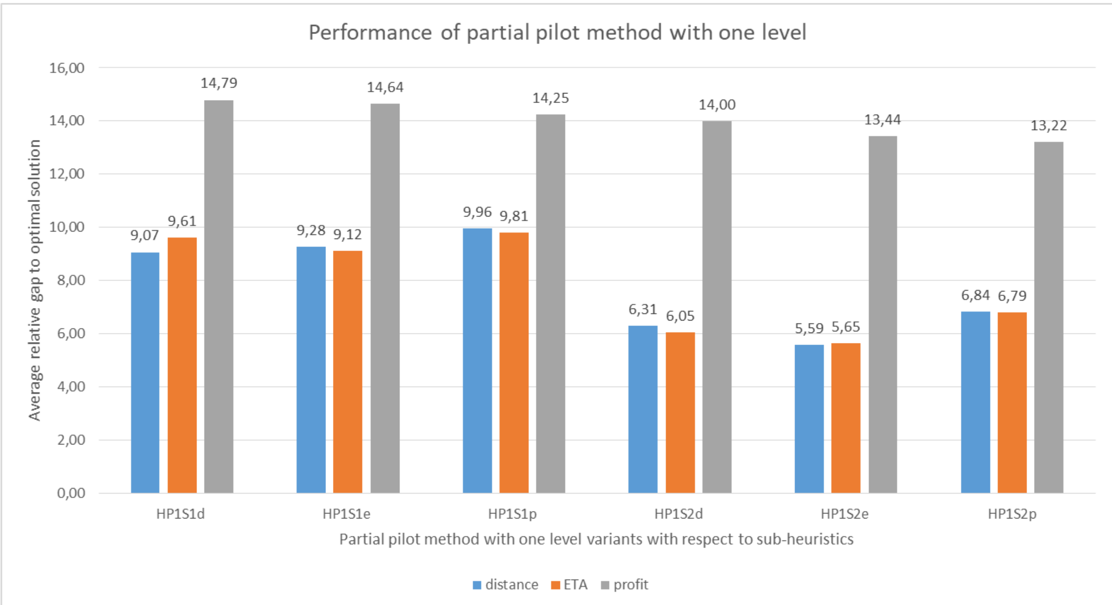

Figure 41. Average performance for partial pilot method variants with one level

The partial pilot method variants display running times that are acceptable for the larger instances tested. The partial pilot methods that were tested correspond to a “minimal pilot method” since the number of assignments evaluated at each level is only two. It would be interesting to test other partial pilot methods with more levels and also with more assignments evaluated in each level and assess the size that these variants would solve given a certain amount of time.

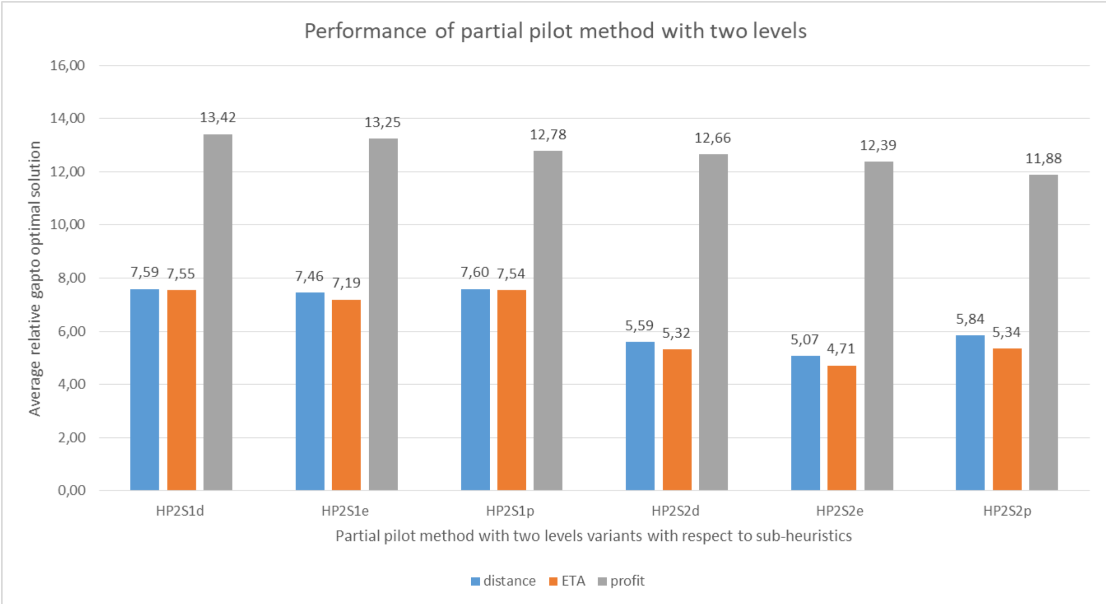

Figure 42. Average performance for partial pilot method variants with two levels

Random cost sets

For the real costs sets, the heuristis and pilot methods that are based on the distance and ETA merit functions and choosing criteria performed quite well when compared to the heuristics that used profit merit functions or profit as a choosing criteria. The purpose in testing the same heuristics and pilot method variants is to investigate if this relation still holds or not.

Table 21 shows the performance for the constructive heuristics where the instances sets have random profits. The average gaps show that the distance and ETA merit function do not provide adequate guidance when choosing the assignments to be added to the partial solution being built. Oppositely and to no surprise, the profit merit function performs much better than the distance and ETA merit functions.

Table 21. Stand-alone results for constructive heuristics

size			HC1d		HC1e		HC1p		HC2d		HC2e		HC2p	
k	ns	np	a%	m%	a%	m%	a%	m%	a%	m%	a%	m%	a%	m%
	15	30	62,65	79,95	60,48	73,12	23,50	42,26	59,72	72,57	61,20	84,32	24,88	46,69
	20	32	64,24	75,58	65,80	78,19	24,80	35,07	67,06	82,97	66,30	79,83	27,12	45,42
3	30	24	71,51	79,38	69,28	80,50	20,14	35,37	70,02	80,28	69,66	80,16	22,26	29,80
	50	10	71,83	79,54	73,18	77,84	16,98	21,61	70,98	78,35	71,68	80,75	19,37	24,68
	80													
	15	30	65,11	82,62	62,59	74,83	22,80	43,06	65,77	80,94	64,18	79,59	19,42	40,81
	20	30	66,11	80,80	67,03	83,04	23,06	33,29	67,97	81,56	66,34	78,61	23,57	34,45
4	30	24	68,28	77,65	69,09	83,91	20,02	29,00	70,39	79,71	69,19	83,34	19,48	31,36
	50	10	73,82	85,95	73,93	87,60	28,25	47,25	73,46	83,85	73,38	84,59	27,00	45,07
	80	5												
	15	30	65,35	85,37	64,14	79,57	24,93	43,49	65,04	82,77	66,03	83,21	17,25	49,78
	20	30	68,33	77,20	69,13	79,07	28,08	37,81	68,60	87,95	68,68	86,07	21,84	36,13
5	30	24	68,31	76,41	70,05	76,53	21,51	27,38	68,10	76,55	68,41	80,49	20,21	31,41
	50	10	72,32	79,90	72,03	84,77	21,01	32,64	72,06	79,61	70,12	82,63	16,43	19,55
	80													
average			68,15	80,03	68,06	79,92	22,92	35,68	68,26	80,59	67,93	81,97	21,57	36,26

Table 22 shows the performance of the full pilot method with one level. Due to the high running times of the pilot method with two levels, these variants were not tested among all the sets of the random group. The profit merit function in both types of constructive heuristics achieves higher performances than the distance and ETA merit counterparts when used as pilot heuristics. These results show that the merit functions based on distance and ETA may not prove a good option if the objective function is not time-dependent. The performance of the full pilot method with HC1p and HC2p as pilot heuristics did not perform so well as in the real cost sets (see Table 9 and Table 22).

Table 22. Performance of full pilot methods variations with one level

size			HC1d		HC1e		HC1p		HC2d		HC2e		HC2p	
k	ns	np	a%	m%	a%	m%	a%	m%	a%	m%	a%	m%	a%	m%
	15	30	29,30	44,71	28,04	45,91	8,64	18,90	26,72	39,57	27,65	45,69	10,95	21,96
	20	32	36,41	47,72	37,48	50,60	12,27	22,44	34,78	49,71	35,58	45,87	19,77	28,42
3	30	24	41,55	55,50	41,68	47,68	11,47	18,43	40,00	49,37	40,83	50,04	12,56	20,74
	50													
	80													
	15	30	28,04	40,85	29,66	44,60	8,14	21,72	26,18	37,04	28,40	43,31	7,88	17,55
	20	30	35,38	47,48	35,06	45,51	12,56	22,23	35,04	51,53	34,15	43,88	15,92	26,65
4	30	24	38,62	46,34	38,78	52,39	10,75	16,12	37,90	46,61	37,74	48,69	11,48	20,73
	50													
	80													
	15	30	26,92	45,17	26,25	51,47	9,95	18,12	28,91	43,34	28,07	46,65	7,06	18,29
	20	30	32,89	44,36	34,00	50,12	13,28	20,21	34,48	44,85	35,24	50,83	13,95	27,50
5	30	24	37,23	44,34	36,06	45,07	13,24	21,83	36,84	48,27	37,45	52,27	9,51	17,77
	50													
	80													
average			34,04	46,27	34,11	48,15	11,15	20,00	33,43	45,59	33,90	47,47	12,12	22,18

Figure 43 resumes the performance of the cosntrcutive heuristics and the full pilot method variants with one level.

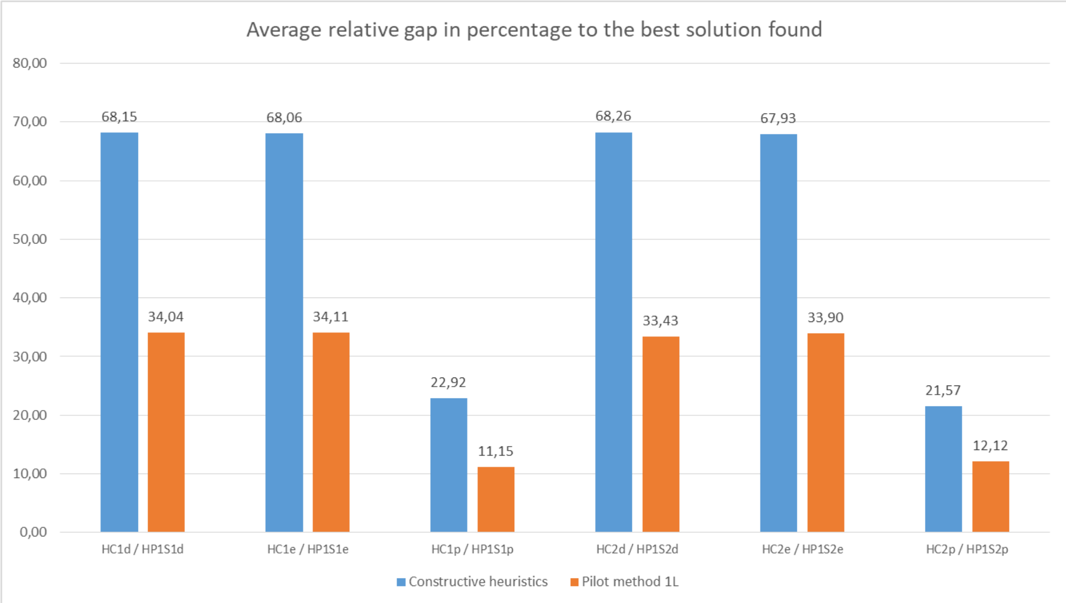

Figure 43. Performance of full pilot method variants and constructive heuristics for random costs sets

Real cost sets with time limit

The following results attempt to answer the question on how the pilot variants perform when there is a time limit to achieve a feasible solution. Only the largest problems where considered (v3n80, v4n80 and v5n80 sets) because for the other problem sets the full pilot variants where able to find a solution within an acceptable maximum time. With limited time, the full pilot method delivers the best pilot solution found.

Table 23. Performance of full pilot method variations with one level and 20 minutes time limit

			HC1d		HC1e		HC1p		HC2d		HC2e		HC2p	
set	np	t	a%	m%	a%	m%	a%	m%	a%	m%	a%	m%	a%	m%
	5	1	17,79	23,10	18,35	21,02	31,95	36,10	15,31	24,05	16,08	20,83	30,81	34,33
	5	5	16,61	21,99	16,65	18,67	28,80	32,45	13,94	20,57	15,01	20,18	30,20	33,14
v3n80	5	10	15,83	20,97	16,11	18,25	26,65	31,79	13,58	19,93	13,72	17,20	28,96	32,22
	5	15	15,49	20,97	15,92	17,97	25,15	30,92	13,39	19,93	13,09	17,13	27,55	30,27
	5	20	15,38	20,71	15,35	17,78	22,69	25,98	13,29	19,93	12,59	16,36	26,21	27,54
	5	1	16,05	17,85	16,67	17,92	26,42	28,48	15,33	18,53	16,08	18,02	26,70	28,46
	5	5	14,16	14,85	14,70	15,51	23,75	25,29	12,65	14,30	12,87	14,10	25,79	28,46
v4n80	5	10	13,52	14,62	14,17	15,41	21,64	23,80	11,56	12,43	12,00	13,65	23,92	25,80
	5	15	13,23	14,62	13,75	15,33	20,46	23,73	11,50	12,26	11,52	13,61	22,47	23,97
	5	20	13,23	14,62	13,46	14,97	19,10	21,84	11,44	12,26	11,43	13,61	22,09	23,55
	5	1	8,71	10,23	9,56	11,43	16,61	20,09	7,55	12,70	8,57	12,44	16,38	18,94
	5	5	7,00	8,55	8,59	9,60	14,18	16,98	5,32	6,96	6,58	9,25	15,71	18,94
v5n80	5	10	6,22	6,95	8,46	9,60	12,84	14,71	4,77	6,09	6,11	9,25	14,78	18,94
	5	15	5,95	6,95	8,06	8,47	11,66	13,90	4,46	6,06	5,85	9,25	14,23	17,27
	5	20	5,59	6,95	7,57	8,13	10,70	13,42	4,25	5,49	5,63	9,25	13,72	15,61

Table 24. Performance of full pilot method variations with two levels and 20 minutes time limit

size			HC1d		HC1e		HC1p		HC2d		HC2e		HC2p	
set	np	t	a%	m%	a%	m%	a%	m%	a%	m%	a%	m%	a%	m%
	5	1	23,20	32,66	21,88	26,05	35,41	39,79	19,78	28,18	20,14	24,90	34,49	37,21
	5	5	19,87	27,68	19,68	23,34	32,23	36,05	16,94	24,02	17,81	24,04	32,40	35,67
v3n80	5	10	17,76	22,92	19,32	23,34	31,68	35,80	16,84	24,02	17,56	24,04	32,04	34,34
	5	15	17,76	22,92	18,06	21,42	31,64	35,72	16,20	24,02	16,78	24,04	31,56	34,34
	5	20	17,47	22,92	18,01	21,42	30,96	35,19	14,78	21,16	15,70	20,19	31,22	34,34
	5	1	24,26	26,78	25,64	30,58	31,75	32,66	22,92	28,03	24,06	30,82	32,26	34,04
	5	5	17,13	19,76	20,09	24,23	27,29	28,85	21,40	25,31	21,96	29,25	30,94	32,26
v4n80	5	10	15,72	17,12	17,44	20,87	26,78	28,40	16,16	19,91	16,90	23,02	27,90	28,93
	5	15	15,71	17,06	15,79	18,21	26,78	28,40	15,52	19,91	16,05	21,36	26,38	27,13
	5	20	15,08	15,89	15,51	16,84	26,78	28,40	15,47	19,91	15,68	21,36	26,26	27,13
	5	1	17,21	20,68	16,66	21,01	21,46	23,20	15,47	20,47	15,58	23,21	21,48	23,58
	5	5	10,04	12,47	12,71	15,73	17,67	19,06	13,35	17,20	14,74	20,80	20,60	22,17
v5n80	5	10	8,07	9,45	10,45	14,53	16,31	19,01	9,25	14,05	10,49	15,82	18,11	19,82
	5	15	7,71	9,45	8,91	9,66	16,23	19,01	7,63	12,01	9,00	15,82	16,76	17,88
	5	20	7,71	9,45	8,70	9,28	16,23	19,01	7,22	11,33	8,48	13,86	16,37	17,88

The superior performance of the full pilot method with two levels over the full pilot method with one level observed in the real costs sets does not hold when time is limited. In all three sets, the full pilot method with one level obtained better performances than the respective counterpart with two levels. We believe that this difference in performance is caused by the number of pilot solutions that are analysed within the same amount of time. The “two level” scheme requires more time to create feasible sequences of two vehicle/survivor assignments that are going to be added to the master solution and obtain the respective pilot solution via the pilot heuristic. With the “one level” scheme, the process of obtaining pilot solutions is more simple and requires less time, thus more pilot solutions are evaluated. Figures 44, 45 and 46 show the comparison of the performance for the constructive heuristics and 20 minutes of the one and two level full pilot methods.

Figure 44. Average performance for full pilot method variants with time limits for real set v3n80

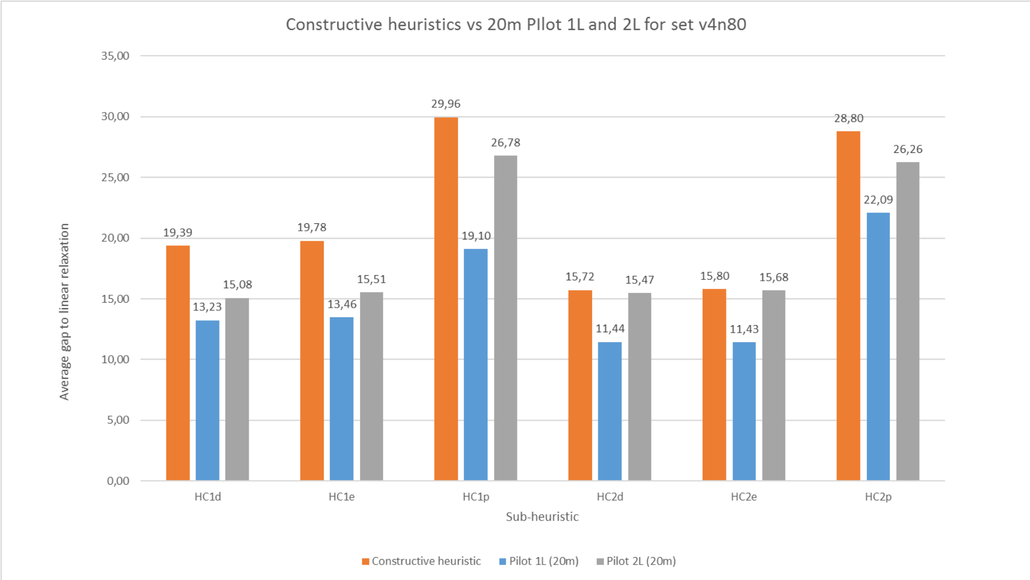

Figure 45. Average performance of full pilot method variants with time limit for set v4n80

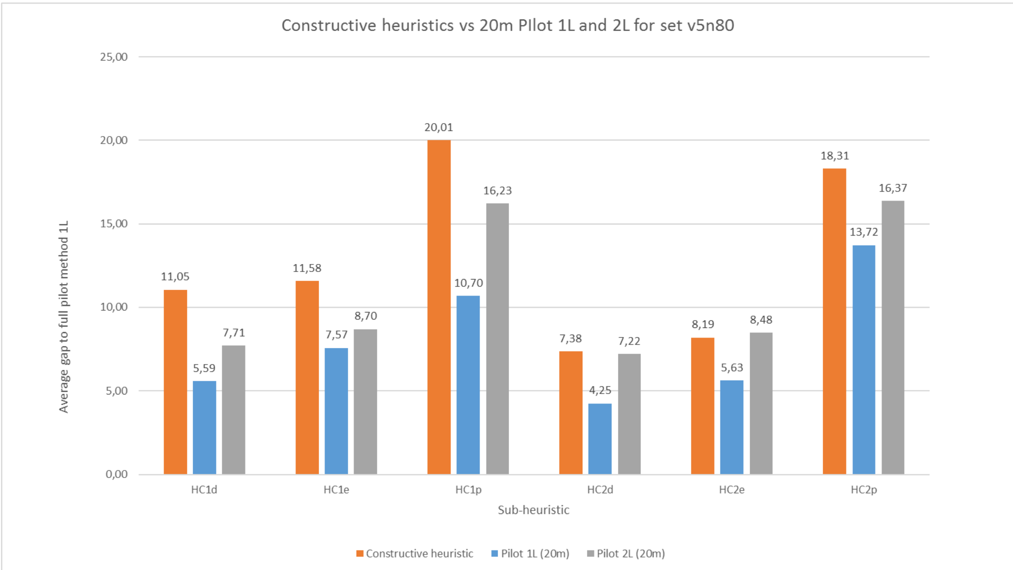

Figure 46. Average performance of full pilot method variants with time limit for set v5n80

With 20 minutes as the maximum time available, the full pilot method with one level performed better than the constructive heuristics. But one may ask if, for the same amount of time, the full pilot method would perform better than the constructive heuristics if larger instances were considered. The answer for this question requires further computational experiments.

3.7 Summary

This Chapter provided a detailed description of the MMRO model and its vehicle flow formulation based on a layered graph. The model can be interpreted as a generalization of the GVRP model proposed by Ghiana and Improta [3].

The first Section covered the construction of the layered graph that represents possible tours made by vehicles that operate to retrieve objects from the water. The size of the layered graph can be quite large even for a relative small number of SAR objects and vehicles. The time step is an important parameter that greatly influences the size of the layered graph and the size of the MMRO problem. The layered graph approach to build the vehicle flow formulation presents two benefits: first, it eliminates the necessity to include constraints that guarantee that vehicles move between time-indexed nodes within feasible time ranges according to their speeds and location in time; second, it eliminates the necessity of constraints to avoid unfeasible subtours. The vehicle flow formulation is based on a binary linear programing formulation and instances with a relative small number of vehicles and SAR objects can easily achieve thousands or millions of variables. Thus, solving the MMRO problem with exact methods can be quite a difficult challenge.

The third Section describes two types of constructive heuristics that build a solution sequentially, step by step, adding in each iteration a feasible vehicle/survivor assignment to the solution being built. The second heuristic is greedier then the first one, since it chooses the most favorable feasible assigment among all possible assigments in each iteration. The choice of the assignment to be added to the solution is made by different merit functions. Some of the merit functions can be understood as the “standard procedure” to define the priority of objects to be retrieved from the water by the vehicles. Distance and ETA can be used to decide the priority associated to an assignment and thus be used as a merit function in the two types of constructive heuristics.

The pilot method is studied using a scheme where sequences of assignments are evaluated when added to the master solution instead of adding only one assignment in each iteration. The “full” pilot method variants tested for the MMRO problem have a high computational complexity. For this reason, several variants were designed that limited the number of assignments evaluated in each level or used a limited amount of time.

The fifth Section covered the main features of the prototype developed to build the MMRO instances and evaluate and compare the quality of different solutions obtained by different algorithms.

The last Section provides computational results for the constructive heuristics and pilot method variants. These heuristics were tested in two groups of MMRO instances: one that contained “real costs” (or real profits), in a sense that the profit associated with each SAR object is decreased with time and is related with the survival times. The second group of instances is obtained from the “real costs” group where the profit is randomly changed for each SAR object. Due to the high computational complexity of the pilot method, strategies are needed to make it pratical and suitable for solving larger instances. The full pilot method variants are not suitable to solve instances with more than fifty SAR objects. With a time dependent objective function where the profit decreases with time, measures based on distance and ETA between a vehicle and an object seem to be suitable to be used as a criteria to choose the assignment to be included in a solution that is being built within a constructive heuristic or a pilot method scheme. Results showed that these measures provide better results when compared to using the profit for the same purpose.

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Chapter 4

Application to the Portuguese Search and Rescue Region

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4 Application to the Portuguese Search and Rescue Region

In this Chapter we use the MMRO problem to estimate the efficacy of the SAR response to a simulated maritime incident based on historical AIS data from a cruise ship transit within the PO SRR and nearby vessels. The incident location is related with the SAR concept of "area remote from SAR facilities" that organizations should identify and map for risk analysis purposes [195].

The Portuguese Navy is responsible for the coordination of search and rescue operations within two search and rescue regions (SRR) within the Atlantic Ocean. Each SRR has a Maritime Rescue Coordination Centre (MRCC) to coordinate SAR operations in response to maritime incidents that may occur within the region's boundaries. The Lisbon MRCC operates within an area that covers most of the Portuguese continental EEZ and part of the Madeira EEZ. Delgada MRCC, located in São Miguel island in the Azores archipelago, operates in the largest SRR of the two and it covers a considerable part of the north Atlantic between the parallels 17º and 45º north.

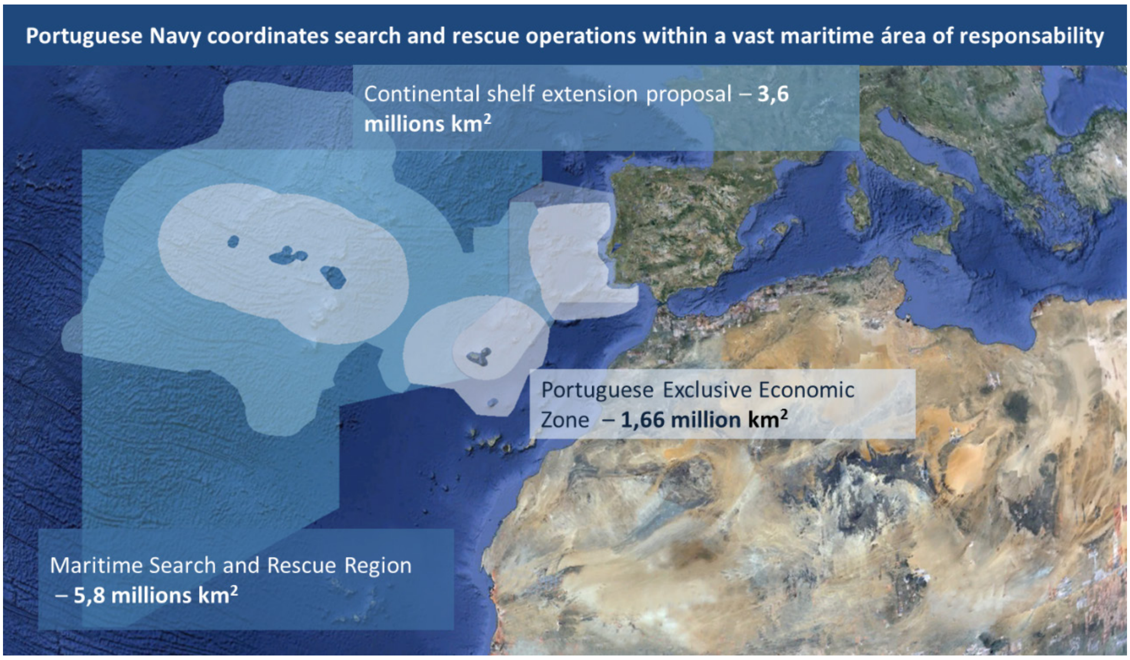

Figure 47. EEZ and SAR areas of Portugal

Recent events in the Middle East and in the north of Africa, especially those involving the activity of organized terrorist groups, and latest's events concerning terrorist attacks in European (EU) cities (see [196]), EU governments have become increasingly concerned with the possibility of maritime terrorist attacks (see [197], [132] and [198]). Refugees from the Syrian conflict (see [199]) and illegal immigrants from North-Africa (see [200]) countries that venture themselves on the Mediterranean to reach European borders remain another source of concern for EU countries which also may require an MRO. Besides the latest apprehensions regarding terrorist attacks, the main causes for high consequence accidents are still related with human error, technical problems and weather conditions [201]. Past maritime accidents show that the number of victims is associated with the type of accident and the ship type. The highest number of mortal victims has been registered in ferry and passenger ships (see [202], [203] and [204]).

Maritime accidents in the portuguese jurisdiction areas are investigated by the Accident Investigation Bureau Maritime and Aeronautical Meteorology Authority (GAMA[30]). The data from these accidents are sent to the European Maritime Safety Agency (EMSA) and they are reported in The Annual Overview of Marine Casualties and Incidents report [205].

In 2015, 884 cruise ships visited Portuguese ports and there were 1.3 million people in transit through Portuguese maritime areas of responsibility (see [206, p. 153]). In 2016, the number of cruise ships totalled 872 which move over 1.2 million passengers (see [207, p. 154]). The three main ports that receive cruise ships are the capital city of Lisbon, the port of Funchal at Madeira Island and the port of Ponta Delgada, in the Island of São Miguel in Azores. The port of Lisbon had a significant increase in passengers numbers of cruise ships between 2015 and 2016 and it is expected that these numbers increase in 2017 due to an increase of 50% in port calls in the port of Lisbon in January compared with the same month in the previous year (see [208] and [209]). Most part of the transit made by the cruise ships that visit portuguese ports is located in ocean areas. Some of the areas are very faraway from SAR facilities and are not frequently crossed by other types of ships. This behaviour is illustrated in the next Section where the concept of "area remote from SAR facilities" is further detailed for the portuguese SRR.

4.1 Areas remote from SAR facilities

Due to the economic value of the cruise ship industry and the elevated risk of terrorist attacks worldwide, the possibility for a maritime incident that may require an MRO has gained attention among national and international authorities. The International Maritime Rescue Federation (IMRF) has produced guidance papers that recommends MRO planners to identify areas, designated as "*areas remote from SAR facilities*", within their overall

[30] GAMA is the acronym for "Gabinete de investigação de acidentes marítimos e da Autoridade para a Meteorologia Aeronaútica".

area of responsibility and prepare a plan to cope with a possible MRO. To classify these areas as "remote areas" one should take into considerations several criteria (see Annex B - Criteria for determining "areas remote from SAR facilities"). While some of the criteria may not be easily ascertain, we enumerate those that can be more easily assessed:

- The number of people at risk;
- availability of SAR facilities and other resources which may be deployed in order to contain the incident and keep those at risk on board until rescued;
- the total recovery capacity of SAR facilities available to reach the scene and recover those who have been taken to survival craft within the five day "time to recover" parameter and/or within survival times;
- the distance (in time) between individual SAR facilities start points and the scene of the emergency;
- the prevailing weather conditions, both on scene and encountered by SAR facilities proceeding.

Although there is no standard "one size fits all" procedure to classify an area as "area remote from SAR facilities" and to what extent it is remote, within the PO Navy, this issue is currently being studied at the PO Naval Research Centre [105]. The purpose is to focus exclusively on cruise ship transits within PO areas of responsibility and perceive where and when those transits occur. It is also important to detect situations or time periods where, during those transits, the cruise ships have no nearby vessels within a certain distance. In this dissertation several areas were considered and the shipping routes and traffic density of AIS equipped vessels was calculated using historical AIS data. The same rational can be applied to any type of vessel, although the number of persons at risk vary significantly. For example, an incident with a fishing vessel that is operating in a remote area far from inland facilities with no nearby vessels will hardly require an MRO. Nonetheless, identifying areas with low shipping density is paramount to ensure a swift assessment of a potential risk that may involve a rescue operation.

Figure 48 shows the density of AIS messages reported by passenger ships equipped with the AIS system during 2016 between Madeira Island and the Morocco coast.

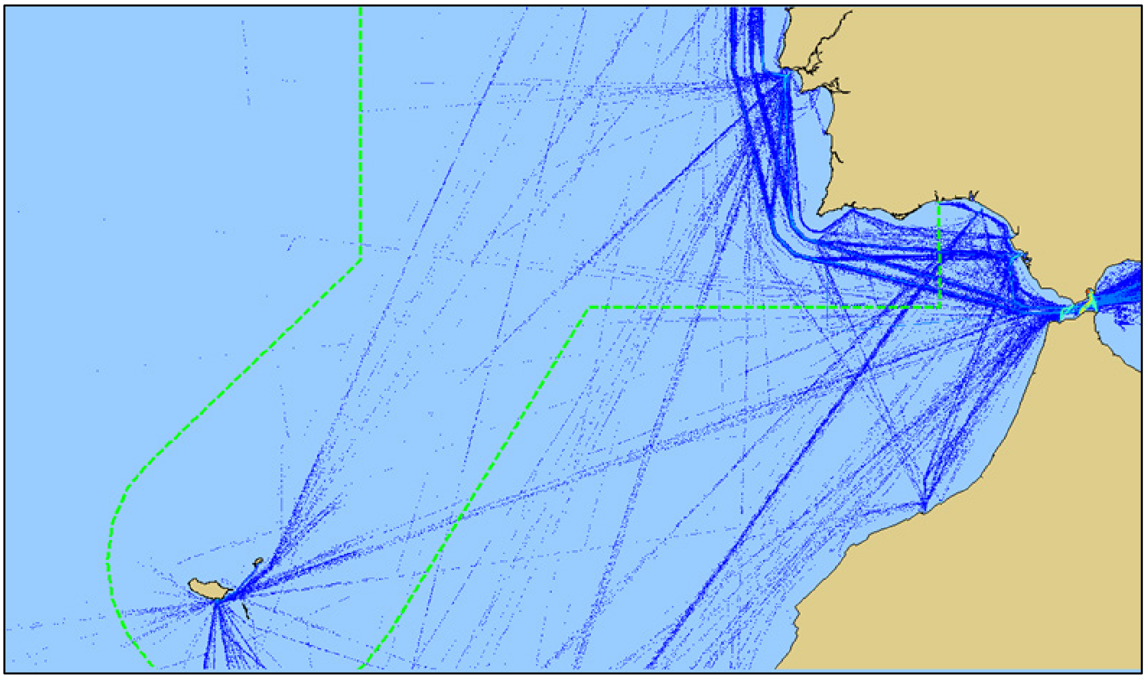

Figure 48. Passenger ship density (period: 2016, mesh: 0.1 Nm^2). Source: PO Navy, Naval Command

Knowing the main routes of cruise ships is the first step to identify the areas where these may be more vulnerable. In [105] an algorithm is proposed to estimate the vulnerability of a cruise ship during its transit within a certain maritime area. The vulnerability is a function of nearby ships and the distance in time to reach the cruise ship location. This algorithm associates a vulnerability value to each position of a cruise vessel during its transit. To do this association it is necessary to list all nearby vessels (regarding their type) to each position of the cruise ship. Such task requires the analysis of huge quantities of AIS data. To perform such analysis a prototype was built in order to apply filters to the AIS data. The filters allow the analyst to choose a specific area (by defining the latitude and longitude coordinates of the respective polygon) and time period. For each defined area it is possible to identify the transits that occur in that area in each day of a certain year. The prototype also calculates a vulnerability index associated to each position depending on the time distance (ETA) of nearby ships. The vulnerability v uses the following formula:

$$v = \begin{cases} 1\ (white) & if\ ETA \leq 1 hour \\ 2\ (green) & 1 < ETA \leq 2\ hours \\ 3\ (yellow) & 2 < ETA \leq 3\ hours \\ 4\ (orange) & 3 < ETA \leq 4\ hours \\ 5\ (red) & 4 < ETA \end{cases} \tag{4.1}$$

In the above formula, ETA refers to the expected time arrival of the nearest vessel. ETA depends on the sustained speed of the vessel. Not always the nearest in distance can be the fastest to arrive at the scene. The vulnerability index shown above is a *myopic* index since it doesn´t consider the number of persons at risk, the total recovery capacity of the nearby ships and survival times.

The figure bellow shows seven polygonal areas, which were designed to study cruise ships transits that visit the port of Funchal in Madeira Island. For the seven defined areas, AIS data was analyzed in order to identify transits of cruise ships in each day of the year of 2016 and surrounding vessels. The rationale behind the areas definition was that these shouldn´t be too large in size and they should contain the major routes of cruise ships that visited the Funchal port. The vulnerability index associated to each position of the cruise vessels can be used to ascertain if a specific area can be classified as "area remote from SAR facilities" and how much remote it is. This issue is still under investigation in the PO Naval Research Centre.

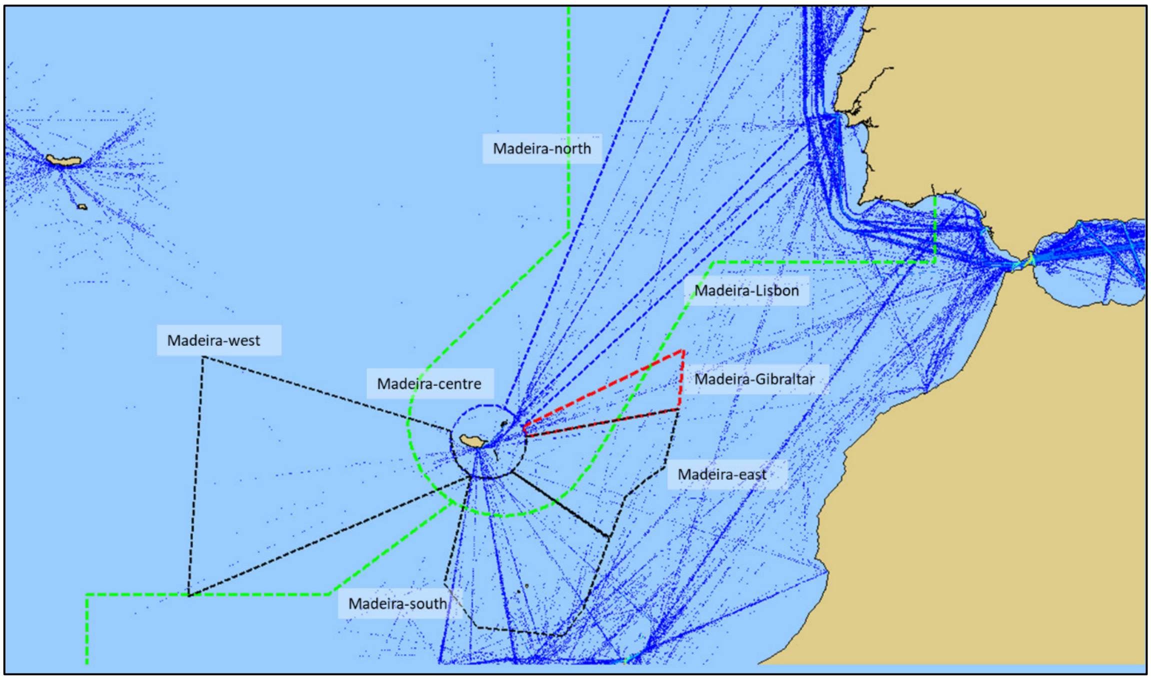

Figure 49. Areas that cover major cruise ship routes that make port calls in Madeira Island

In order to demonstrate how the MMRO problem can be used to estimate the efficacy of a mass rescue operation, we take a transit of a cruise ship and for a specific position we check the surrounding vessels within a fifty nautical miles radius and consider several incident scenarios. The scenarios depend on the number of persons at risk and their respective survival times. The main idea of the vulnerability index is to demonstrate that "remote areas" don't have to be very far away from mainland infrastructures. In the case of mid-Atlantic areas, such as the limits of the eastern Madeira SRR, there are periods of time where assistance can take more than five hours to reach a specific incident location.

4.2 The scenario: incident during transit

On april 27th 2016, the cruise ship Vision of the Seas departed from the Funchal port at 08:00 PM heading to the port of Malaga, in Spain. The transit happened with no incidents and all passengers enjoyed their voyage between Madeira and Spain. Using historical AIS data one can recreate the sea picture during the Vision of the Seas transit and check the vessels that were nearest to the cruise ship at a certain position. The Vision of the Seas is a 78.000 Ton cruise ship that carries a total of 2435 passenger and a crew of 765 persons. It is operated by the shipping company Royal Caribbean. The voyages are planned so that during the day passengers can visit the mainland and do some sightseeing and land excursions. The night period is usually used for transit between ports.

Figure 50 shows the "Madeira-Gibraltar" area (bold slashed red) showing a sequence of positions during the Visions of the Seas transit between Funchal port and Malaga between 27th April 00:00 and 04:41:56 (last position inside the Madeira-gibraltar area). The transit positions correspond to AIS messages sent by the system aboard the ship. The positions were also coloured using the vulnerability index mentioned before. The "green" dot means there is at least one vessel that could reach that position within 2 hours.

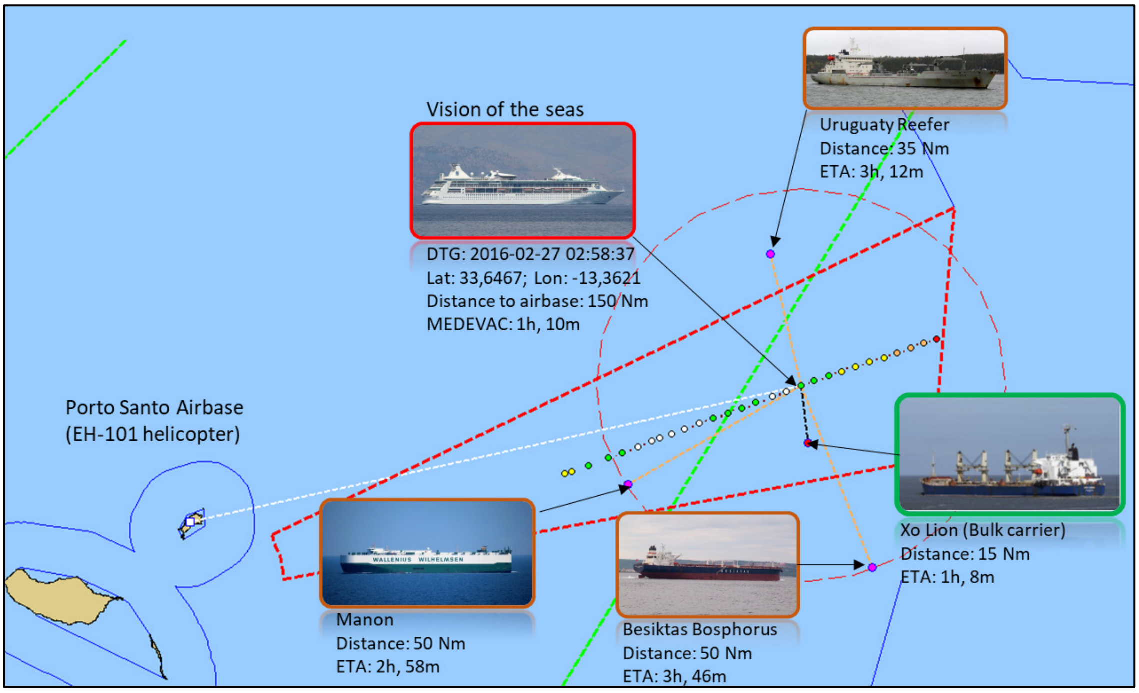

Figure 50. Visions of the Sea transit on April 27th 2016 inside Madeira-Gibraltar area at 02:58:37

What deserves attention are the red dots, since they represent moments during the transit where assistance would take more then four hours to reach that position.The figure also shows a circle centered on a selected position with a fifty nautical miles radius (light slashed red). All the AIS equipped vessels within that circle are shown. Four ships were making their course at the time instant of the selected position (02:28:37 AM). The nearest ship is a bulk carrier, named Xo Lion. This small number of vessels allows MRO planners to recognise and be aware that the Madeira's eastern limits of the SRR do not have a high density of marine traffic and that may present a serious challenge if one has to handle with a MRO. Several scenarios can be designed to simulate an incident where a large number of persons are in distress and some of them require immediate extraction to a safe place. We may consider also survival times associated to some of the passengers in which we can assume death if they are not recovered within a certain amount of time. In order to use the MMRO problem to model a specific situation we must take into consideration not only the specific characteristics of the incident but also the dimensions of the problem we are able to tackle. The MMRO problem dimension depends mostly on the number of "objects" to recover and the number of time periods within the overall mission window or mission duration. It is import to note that, any position (dot in the above figure) could be selected to recreate an incident. In the next figure, it is shown the nearby ships within a fifty nautical miles radius from the position at the instant 03:29:26 AM on 27th April.

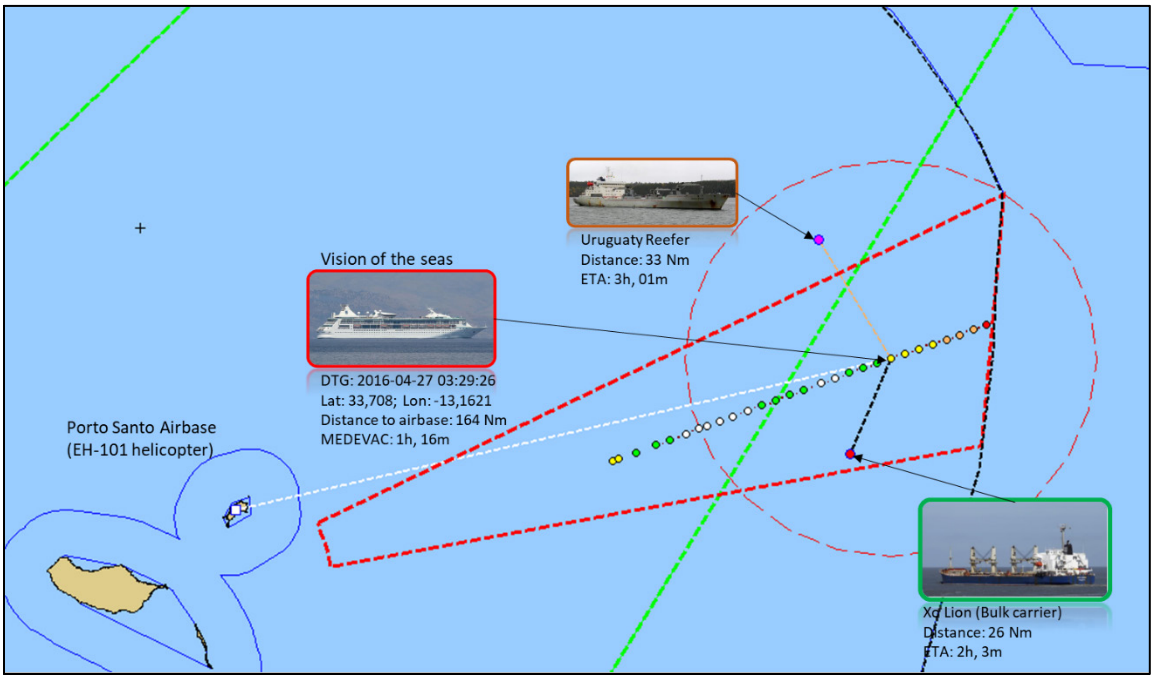

Figure 51.Visions of the Sea transit on April 27th 2016 inside Madeira-Gibraltar area at 03:29:26

The vessels Mano and Besiktas Bosphorus are outside the fifty nautical miles circle (reason for not being shown).

For the purpose of the demonstration we have chosen the position with coordinates 33.708 degrees in latitude and -13.1621 degrees in longitude at time 03:29:26 AM. At that location the cruise ship distances 164 nautical miles from Porto Santo airbase (which has an EH-101 helicopter for SAR purposes).

For this specific location of the cruise ship we are interested to evaluate what could have been done (with respect to rescue operation) if there were an incident in which some of the passenger would be adrift individually or in liferafts and if some of those were injured and their survival time would require immediate recovery.

There are an infinite number of possible situations that can be considered to set the resulting consequences of a maritime accident. Here we are only interested in the consequences or effects that characterize the accident after it has happened and not on the causes behind the accident. The scenario to demonstrate the MMRO problem has the following assumptions:

- The cruise ship has full passenger and crew capacity (3200 persons aboard);
- Cause of incident calls for captain to give order to abandon ship and evacuate all passengers and crew;
- 25% of passengers and crew dye immediately from the incident (780 victims); remaining 75% survives the incident (2420 survivors);
- The surviving passenger and crew (2420 survivors) will be located in:
    - Liferafts (6 liferafts in the water carrying 50 persons each; 300 persons);
    - Lifeboats (14 lifeboats in the water carrying 150 persons each; 2100 persons);
    - Water (20 persons in the water);
- Each person in the water (PIW) will have a survival time associated (time of death is known for efficacy estimation purposes only, not used for rescuing purposes);
- The accident happens at the DTG 02:58:37 AM on April 27th and the alert is given 1 hour later;
- Evacuation of survivors is complete within 1 hour (DTG 03:58:37 AM on April 27th);
- Although the incident is outside the Lisbon SRR, the MRCC Lisbon coordinates the SAR operations;
- Vessels Uruguay Reefer and Xo Lion are called to assist in the recovery of the survivors. Both ships act as SAR facilities and change their course to the incident location;
- Uruguay Reefer can recover 1500 persons on board while Xo Lion can recover 2000 persons;
- Goal: to assess the efficacy of the SAR response in the first six hours subsequent to the incident.

For the above scenario, three variations (variants designated by the letters A, B and C) are considered based on the survival time of the twenty survivors who are in the water. All remaining survivors aboard liferafts or lifeboats have 3 days of survival time. For scenario A, the PIW survival times were randomly generated with uniform distribution between the 04:00:00 AM and 10:00:00 AM, corresponding to the total mission duration of 6 hours. In scenario B we assume higher consequences by considering the survival times between 04:00:00 AM and 08:00:00 AM. In scenario C survival times are randomly generated between 04:00:00 AM and 06:00:00 AM.

4.2.1 Setting the MMRO problem data

The first step to set the MMRO problem data for the above scenario is to define the airbase (depot), the SAR objects and the nearby ships initial location. The location of these elements is set using the GIS functionalities available in the MMRO prototype. The problem's incident has the following data:

- Incident DTG: 2016-04-27 04:00:00
- Total drift duration (mission time-window): 6 hours
- Time step: 5 minutes
- Number of time stamps: 72 (6 hours has 72 periods of 5 minutes each)
- Available vehicles:
    - helicopters located at Porto Santo Airbase
        - Number of helicopter: depends on scenario variant
        - Distance to incident: 164 Nm
        - Cruise speed: 100 Kts
        - Time to recover single PIW: 20 minutes
        - Autonomy: 400 nautical miles
        - Passenger capacity: 15 persons
    - 2 nearby ships (opportunity vessels):
        - Uruguay Reefer
            - Distance to incident: 33 Nm
            - Initial location: 34.1726 latitude and --13.5291 longitude
            - Cruise speed: 10 Kts
            - Time to recover one survivor: 5 minutes
            - Autonomy: no limit
            - Capacity: 2000 passengers
        - Xo Lion
            - Distance to incident: 26 Nm
            - Initial location: 33.4218 latitude and -13.3646 longitude
            - Cruise speed: 10 Kts
            - Time to recover one survivor: 5 minutes
            - Autonomy: no limit
            - Capacity: 1500 passengers
- Meeting location for survivors transfer at coordinates 33.7760 latitude and -13.2993 longitude

- Survivors initial location and estimated survival time:
  - 40 objects considered:
    - 6 Liferafts with 50 passengers each
      - Estimated time alive: 3 days (72 hours)
    - 14 lifeboats with 150 passengers each
      - Estimated time alive: 3 days (72 hours)
    - 20 PIW
      - Location: randomly distributed within 1 nautical mile radius from the centre of the incident
      - Estimated time alive:
        - Variant A: Uniform distribution within 6 hours from incident DTG
        - Variant B: Uniform distribution within 4 hours from incident DTG
        - Variant C: Uniform distribution within 2 hours from incident DTG
- Objective function:
  - Linear decay function assuming the value 10000 at the incidents DTG and the value 500 at the object's death DTG. For instant subsequent to the death DTG the objective function is constant with the value 500.
- Rescue plan
  - Heuristic: Constructive Heuristic, greedy variant, with ETA as merit function

The next figure shows the MMRO prototype with the above scenario data and a rescue solution obtained by the heuristic HC2e.

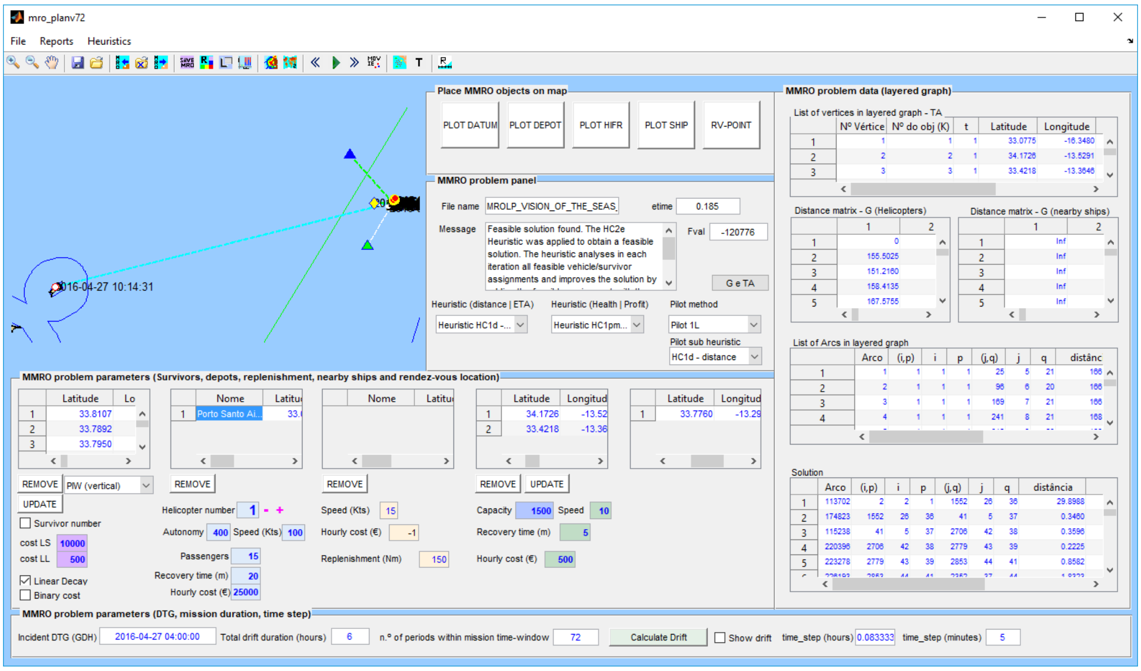

Figure 52. MMRO Prototype with incident data and rescue solution

The greedy constructive heuristic with ETA as merit function was used to obtain a feasible solution for the MMRO problem. The feasible solution defines a rescue plan where the object to be recovered by a certain vehicle is the one with the lowest expected time arrival (ETA). This means that, at a given moment, if there are two vehicles available to recover an object, the vehicle/object assignment chosen is the one where the vehicle is faster to arrive at the object's location.

4.2.2 MMRO Efficacy

The efficacy of the rescue plan is calculated for each type of SAR object. Since liferafts and lifeboats have a survival time of 3 days, these are easily recovered by the nearby ships, Uruguay Reefer and Xo Lion. In a real situation, the lifeboats and liferafts could be tied to the ship's side to keep them from drifting. Some of the more injured survivors could make to the ship's accommodations for shelter while the remaining could stay on the liferafts or lifeboats. There are a total of 9 variants that result from combining the three survival times for PIW and the number of available helicopters. The variant are designated by the letters "#L", where the cardinal represents the number of available helicopters and "L" refers to the letter associated with the survival times. The variant "2B" corresponds to the scenario with two helicopters and survival times for the PIW ranging between 04:00:00 and 08:00:00. The scenario variants are intended to demonstrate the impact in the overall efficacy from having more available SRU's given a predefined set of effects (consequences) resulting from the incident. The figure below resumes the estimated efficacy for each SAR object within each of the scenario variants considered:

Table 25. SAR efficacy for MRO scenario variants

	Life raft						**Life boat**						**PIW**						**Mission Time**
variant	n	LL	LS	EFF	TT	RT	n	LL	LS	EFF	TT	RT	n	LL	LS	EFF	TT	RT	MT
1A	6	0	6	100%	3 hours, 49 mins	1 hour, 29 mins	14	0	14	100%	4 hours, 34 mins	1 hour, 49 mins	20	10	10	50%	4 hours, 39 mins	2 hours, 59 mins	4 hours, 39 mins
1B	6	0	6	100%	4 hours, 39 mins	1 hour, 14 mins	14	0	14	100%	4 hours, 39 mins	1 hour, 54 mins	20	10	10	50%	4 hours, 39 mins	2 hours, 59 mins	4 hours, 39 mins
1C	6	0	6	100%	3 hours, 14 mins	49 mins	14	0	14	100%	4 hours, 14 mins	1 hour, 29 mins	20	20	0	0%	4 hours, 44 mins	3 hours, 4 mins	4 hours, 44 mins
2A	6	0	6	100%	3 hours, 44 mins	1 hour, 19 mins	14	0	14	100%	4 hours, 24 mins	2 hours, 4 mins	20	9	11	55%	3 hours, 59 mins	2 hours, 19 mins	4 hours, 24 mins
2B	6	0	6	100%	3 hours, 14 mins	49 mins	14	0	14	100%	4 hours, 14 mins	1 hour, 29 mins	20	7	13	65%	4 hours, 14 mins	2 hours, 34 mins	4 hours, 14 mins
2C	6	0	6	100%	3 hours, 9 mins	44 mins	14	0	14	100%	4 hours, 4 mins	1 hour, 4 mins	20	20	0	0%	4 hours, 19 mins	2 hours, 39 mins	4 hours, 19 mins
3A	6	0	6	100%	3 hours, 14 mins	44 mins	14	0	14	100%	3 hours, 54 mins	1 hour, 34 mins	20	7	13	65%	3 hours, 39 mins	1 hour, 59 mins	3 hours, 54 mins
3B	6	0	6	100%	3 hours, 14 mins	44 mins	14	0	14	100%	3 hours, 54 mins	1 hour, 34 mins	20	6	14	70%	3 hours, 39 mins	1 hour, 59 mins	3 hours, 54 mins
3C	6	0	6	100%	3 hours, 14 mins	44 mins	14	0	14	100%	3 hours, 54 mins	1 hour, 34 mins	20	20	0	0%	3 hours, 39 mins	1 hour, 59 mins	3 hours, 54 mins

In Table 25, the total time (TT) represents the elapsed time from the incident's DTG until the last object is recovered, for each type of SAR object. TT is also the time that a certain type of objet spends in the water drifting. The recovery time (RT) is the elapsed time between the first and the last object recovered. RT can be seen as a performance indicator of the rescue operation for a specific type of object. For example, in variant 1A, all liferafts were recovered within 3 hours and 49 minutes from the incident's initial DTG.

All these indicators provide useful data that is not directly obtained from the MMRO solution and may be used to define benchmark performance values for rescue operations. Another important statistic is the overall time required to recover all SAR objects which defines the mission duration. By using a heuristic and observing that a feasible solution only requires a smaller amount of time than the mission duration defined by the user, then the problem can be rebuilt with a shorter mission duration with the benefit of having a lighter problem (regarding the size of the data structures).

The results show that having more helicopters increases the PIW rescuing efficacy. In variant A, which is the more "optimistic" scenario, we observe a 5% increase in the PIW efficacy by adding one extra helicopter and a 15% increase when two extra helicopters are made available. Having extra helicopter also reduces the overall mission time (MT). Given the location of the incident, if more helicopters were to be deployed in rescuing activities, these would have to be dispatched by the Spanish SAR system (located in the Canary islands archipelago) or by the Morocco SAR system.

The following figures show the SAR Efficacy for each type of object and the vehicle load in the final solution for the scenario variants B:

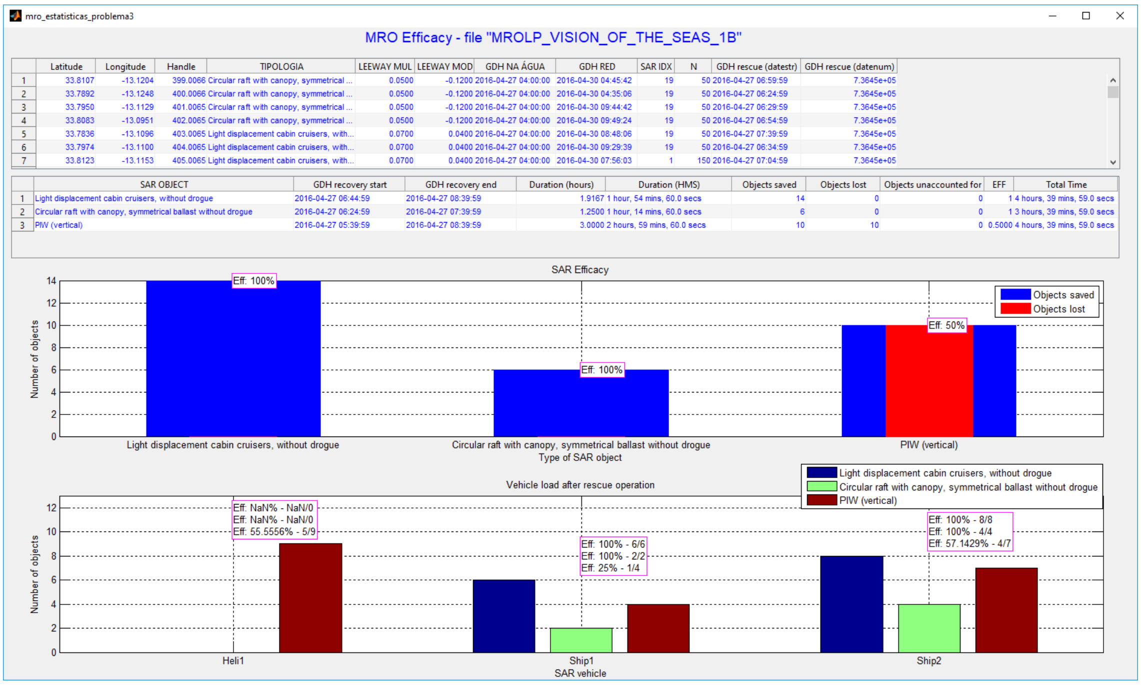

Figure 53. SAR efficacy and vehicle load after rescue operation - variant 1B

In variant B, having two helicopters changes the efficacy of rescuing PIW in 15%. Three helicopters provide an increase of 20% compared to only one. For variant C, the availability of one extra helicopter makes no difference in the operation overall efficacy.

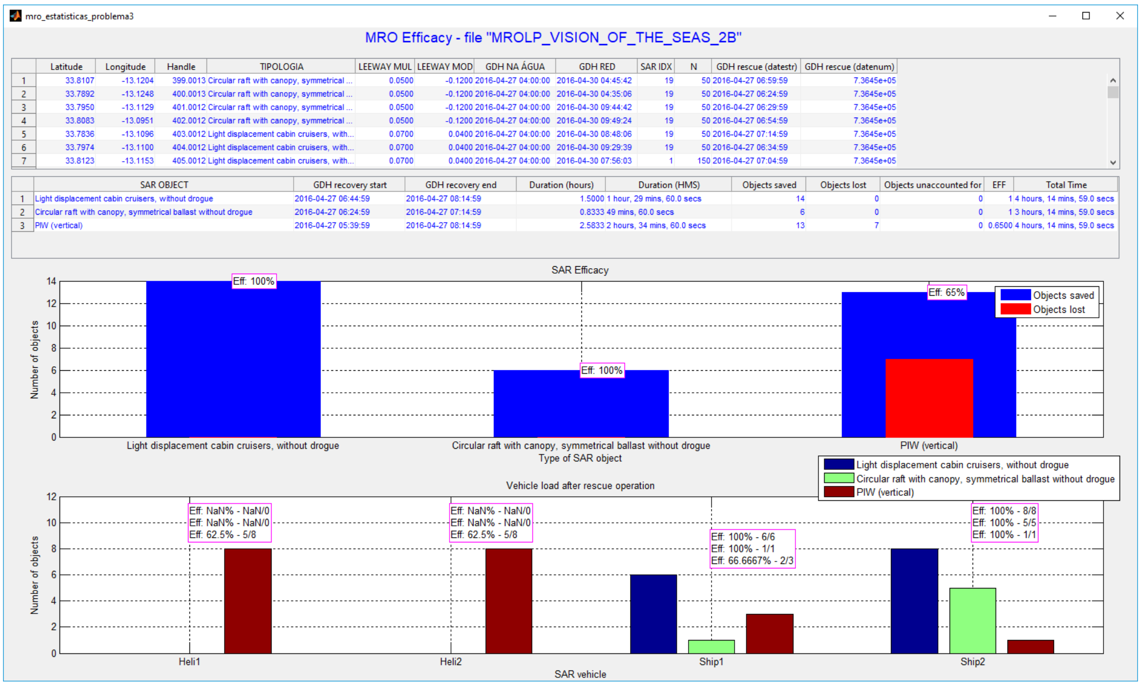

Figure 54. SAR Efficacy and vehicle load after rescue operation - variant 2B

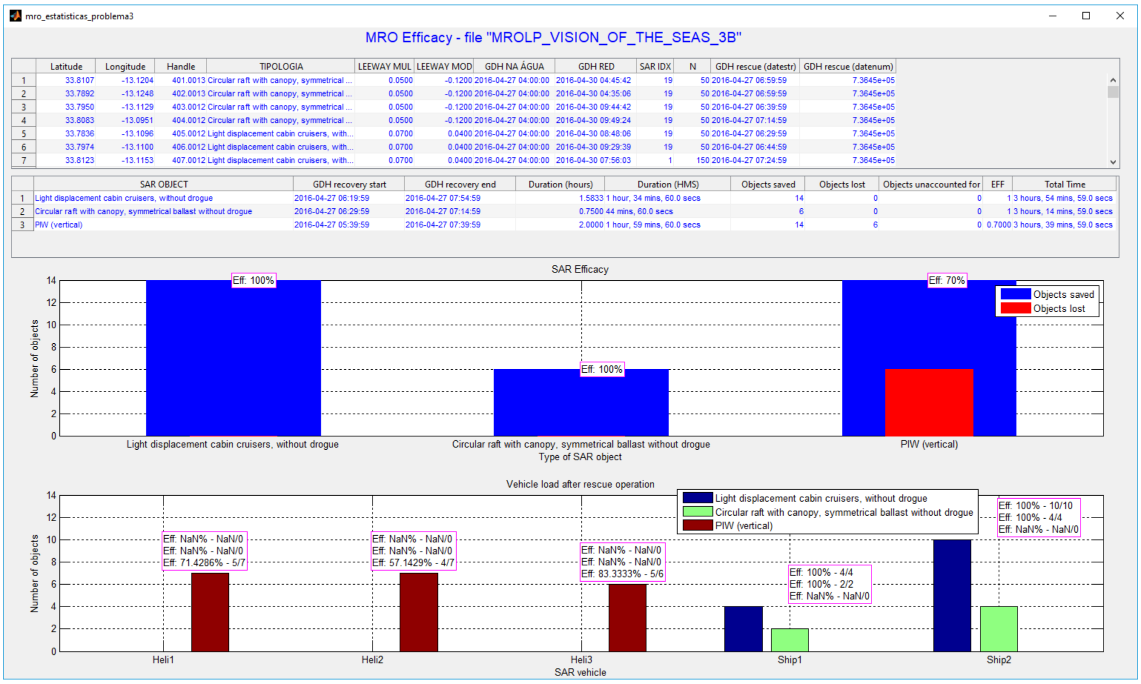

Figure 55. SAR Efficacy and vehicle load after rescue operation - variant 3B

It is interesting to note that with three helicopters, the nearby vessels do not recover any PIW. “Ship2” corresponds to Xo Lion, who is the nearest ship from the incident’s location. Since it is the nearest ship it is natural that it will recover more objects since it arrives earlier to the scene.

The scenario variants provide some insight of a possible outcome for MRO involving the three ships and available helicopters in the Porto Santo airbase. Some of the statistical indicators are agnostic to the consequences considered in the scenario variants: for example, the travel time each nearby vessel takes to reach the scene does not depend on the consequences of the incident. The travel time of each nearby ship can be reduced if speed is increased. Normally merchant vessels navigate at cruising speed which is optimized for an acceptable compromise of fuel efficiency and travel time between port calls. The urgency of the situation may need that nearby ships make top speed to the incident location.

The MMRO instances tested in this Section show that the efficacy of the rescue operation depends largely on the survival times of the persons involved. The scenarios variants contribute to map a specific SAR capability to a maritime area where cruise ships pass through. The "SAR capability" depends considerably on the survival times, nearby ships recovery capacity and their time distance to the scene. Weather conditions are used for drift calculation purposes but they may also be used to condition the parameter that specifies the time required for a ship or helicopter to recover a SAR object. For the scenario variants that have only one available helicopter, the estimated efficacy corresponds to the current "normal SAR capability" evaluation (conditional to the survival times considered). Although the area where the cruise ship passed is within the range of the SAR helicopter, in case of a high consequence incident, specific locations related with the cruise ship transit can be considered as having an enhanced risk regarding low efficacy of an MRO due to low density shipping nearby. Such locations should be considered as "areas remote from SAR facilities".

4.3 Procedure to characterize areas remote from SAR facilities

The previous Section shows how to construct MMRO instances with the prototype tool that takes into consideration the position of a cruise ship, nearby vessels and the weather conditions at a specific moment and relates it with an incident with several types of SAR objects involved with their respective survival times. The efficacy associated to the solutions obtained for each instance can be used to characterize the maritime area crossed by cruise ships. The calculations of ETA between nearby ships and a cruise ship location at a specific moment in time can also be used, exclusively by itself, to define the vulnerability of that cruise ship at that specific location and time. The concept of vulnerability has multiple interpretations depending on the context where it is used. In the context of this dissertation, vulnerability refers to the efficacy of the SAR response to an incident that requires an MRO. A cruise ship during a transit is more vulnerable if, in case of an incident that would require an MRO, the SAR system cannot cope with the means to respond effectively and guarantee a certain degree of success in the rescue operations. If no more information is available, the ETA can be used to estimate the vulnerability associated to a cruise ship in case of an incident that requires a MRO. In [105] Nascimento uses ETA to measure the vulnerability associated to cruise ships during their transit and studies a series of maritime polygons in order to assess which areas should deserve more attention by the Portuguese SAR system. The idea is to provide information, based on historical AIS data, of specific maritime areas that are crossed by cruise ships and the time required to assist them, which may takes several hours. Using AIS data from 2016, the primary result of this work were a series of vulnerability maps within the PO SRR for cruise ships. Predictably, polygons near shore (islands in archipelago or continent shore) present low vulnerability values due to the high density of maritime traffic (see Annex C - Vulnerability maps for cruise ships).

Figure 56 depicts the passenger ships AIS density between January 01 and december 31 of 2016. Figure 57 shows the vunerability index expressed in (4.1) over the passenger ships density. Both figures were produced with the vunerability algorithm described in [105].

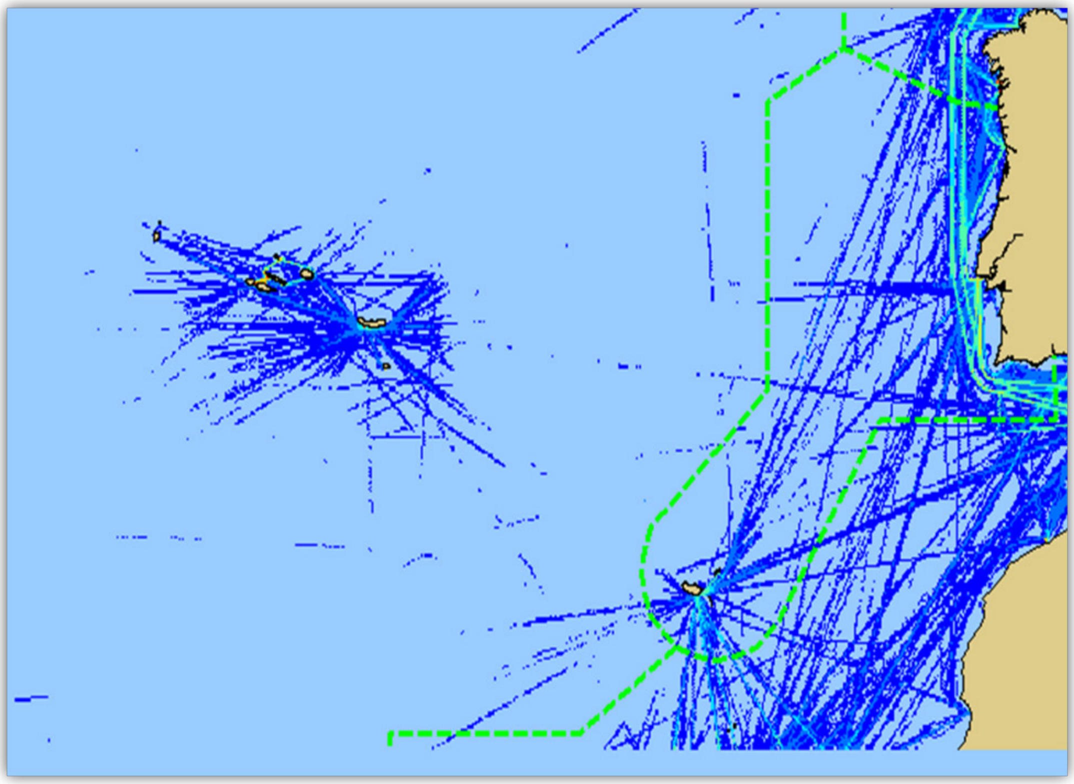

Figure 56. Passenger ship AIS density. Period: 2016. Mesh size: 5 Nm^2

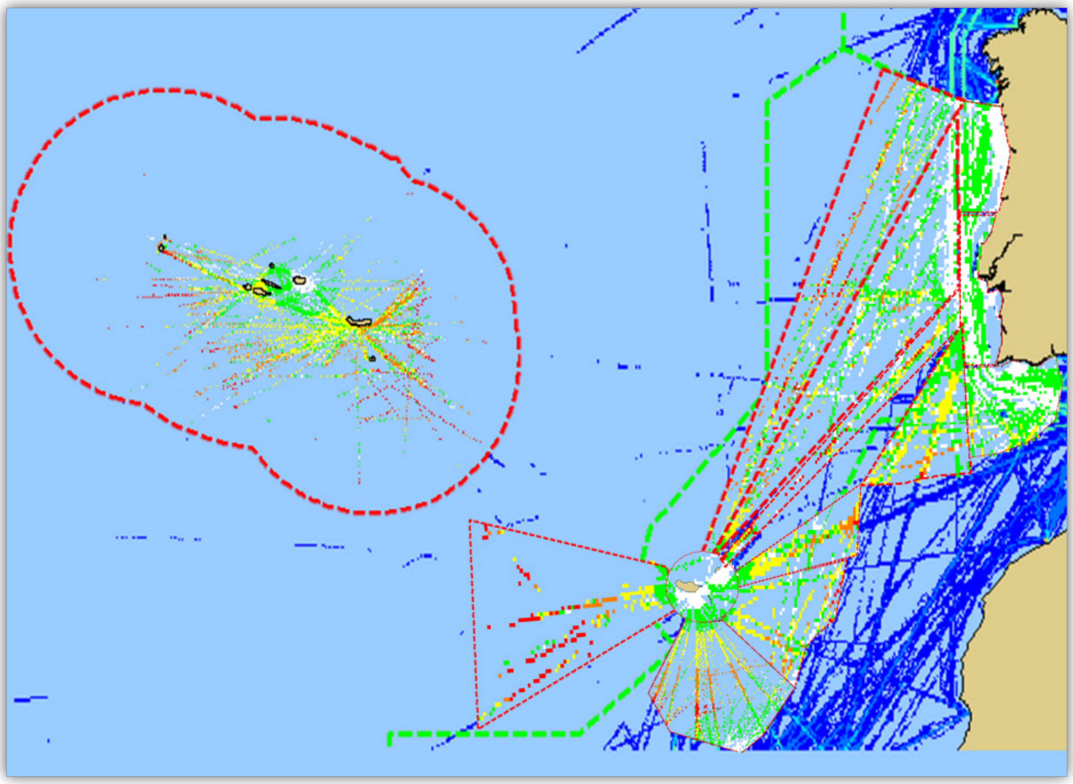

Figure 57. Vulnerability index (4.1) over passenger ship density. Mesh size: 5 Nm^2

The vulnerability function expressed in equation (4.1) and used to build the density maps in Figure 56 and 57 does not take into consideration the recovery capacity of nearby ships. This function can be improved by considering the amount of recovery capacity that can be deployed to the scene within a certain amount of time. As it would be expected, any function to measure vulnerability needs to be validated by experts to guarantee that the provided information is usefull for assessing the capability of the system to respond to an incident in a specific location and type of distressed vessel. For example, taking R as the ratio between the total passenger capacity of nearby ships and the total number of passengers of a cruise ship (potencial number of survivors to be rescued), an improved version of the measure in (4.1) can be expressed by:

$$v = \begin{cases} 1\ (white) & R \geq 1\ within\ 1\ hour \\ 2\ (green) & R \geq 0.8\ within\ 2\ hours \\ 3\ (yellow) & R \geq 0.5\ within\ 3\ hours \\ 4\ (orange) & R \geq 0.5\ within\ 4\ hours \\ 5\ (red) & otherwise \end{cases} \tag{4.2}$$

In a situation where there is the need to retrieve one thousand persons that are in the water and there is only a fishing vessel nearby that can reach the scene in less than one hour, the vulnerability associated with the cruise ship cannot be low or take the value 1 (lower is better). A fishing vessel, depending on its size, can retrieve only a few dozens of survivors without compromising the safety of the ship's crew. With the measure in (4.2) the capacity of nearby ships is taken into consideration to assess the available resources (in this case, the resources correspond to facilities such as cargo ships or other types of vessels) that the SAR system may assign to the scene.

The vulnerability index in (4.2) is simply an example of what can be used to assess whether a cruise ship is crossing a maritime area where the SAR system would have difficulties in providing assistance or, to characterize a maritime area based on the average of the indexes calculated for each position of the cruise ships that crossed that area in a certain time period. This index does not reflect the severity of the possible effects from an incident taking place at a specific moment and location. These effects would account for possible number of victims and survivors with their own survival times. Instead of changing the (4.2) index, one could use the MMRO problem to create instances and use the efficacy of the solution and associate it to the vulnerability index. Thus, for each position of a cruise ship one may have the tuple (v, eff). As expected, this approach would require the definition of scenarios where the possible effects of an incident (that would require an MRO) would have to be categorized into variants with increased complexity and severity (similar to the approach followed in Section 4.2). Given the large number of situations that may be considered, using the MMRO problem to assess the possible outcome of an MRO for each position of the trajectory of a cruise ship would lead to a very time consuming process. The results obtained by such approach would only be valid if the trajectories of the cruise ships would remain unchanged in the future as well as the maritime traffic in the area. Fortunately, maritime routes do not change significantly during consecutive years and if such study would be carried out, the results could provide valid knowledge for specific maritime areas for the upcoming years.

The great number of MMRO instances that would have to be created to characterize a maritime area could be diminished if a limited number of cruise ship positions were selected and also by selecting a small number of variants for hypothetical MROs. For example, for a specific maritime area, during a year, one could choose positions of cruise ships that are one hour (or thirty minutes) apart to create two categories of MMRO instances. These categories could differ in the number of survivors in the water (PIW) relatively to the number of liferafts or lifeboats. Given the size of the MMRO instances that can be made, the total number of SAR objects should not exceed one hundred. We propose to consider two different scenarios where there are no mortal victims but all passengers are required to abandon ship (for example, a fire onboard that rapidly becomes out of control) and they differ on the number of persons in the water. Both scenarios should not exceed a total number of 100 SAR objects, due to the size of the MMRO problem data structures.

Two scenarios can be considered for an hypothetical incident that would require an MRO:

1. Scenario A. All passengers are aboard liferafts and lifeboats, which will drift apart due to the maritime drift;
2. Scenario B. All passengers are aboard liferafts and lifeboats except a small number of persons that are in water (not larger than 50 PIW), which will drift apart due to the maritime drift.

Both scenarios are pessimistic (most dangerous scenario) concerning the effects of the incident. The incident with the Costa Concordia and the scenarios share the fact that passengers had to abandon the ship. Worst scenarios than A and B can only contemplate mortal victims associated with the incident. But these scenarios present the highest potential to damage the credibility of the SAR system. In this case, since the efficacy of the rescue operation depends on a large number of lives that require assistance after notification, if they are not rescued alive, it may drop the overall efficacy indicator significantly. If there were mortal victims prior to the notification, those numbers would not be accounted for the efficacy indicator (see **System effectiveness and efficiency** in subsection 2.1.3) and the overall efficacy indicator would not be affected by this number. The most dangerous scenario where there are no mortal victims after an incident is the one where all passengers have to abandon the ship. For scenario A, since all passengers are aboard liferafts or lifeboats, we can consider that the survival time associated to each passenger is 7 days (one week). This value can be changed within the prototype while creating the MMRO instance. As for scenario B, it is important to define the survival times of the PIW. Since we are interested in evaluating the SAR response within the first six to eight hours, then the survival times for the PIW can be randomly generated between that interval of time. Although scenarios A and B have similarities with the Costa Concordia incident, the purpose is to set such type of incident within ocean waters where help may take some time to reach the scene.

The following procedure describes the major steps to create vulnerability and efficacy maps based on the index described in equation (4.2) and efficacy maps based on the solutions of the MMRO instances built according to scenarios A and B.

Procedure 2 – Efficacy index for maritime areas

Inputs: Set of polygons P (set of maritime areas), Time window T, Available AIS data, mesh size (nautical miles), time distance between cruise ship positions (minutes), Constructive heuristic HC2, Algorithm 3.1.

Output: vulnerability and efficacy maps for the set of polygons P

For $k = 1 \colon P$ (maritime area k)
    Produce list L with all cruise ships positions within time window T
    For $i = 1 \colon L$ (position i in list L)
        Build list N_i with nearest position of nearby ships relative to position i
        Calculate vulnerability index v associated to position i
        Build MMRO instance using algorithm 3.1 for scenario A and B
        Calculate eff for MMRO instance A and B associated to position i
    Build the vulnerability map for polygon k
    Build the efficacy map variant A and B for polygon k

The above procedure considers a set of different polygons within a certain maritime region of interest. It is important to consider a maritime area (defined by a polygon) where it is known to be traversed by cruise ships. Otherwise, it makes no point in calculating the vulnerability or efficacy. The vulnerability maps presented in "Annex C - Vulnerability maps for cruise ships" are built using the *surfacem*[31] function available within MATLAB's Mapping Toolbox. For each polygon $k \in P$ there will be two matrices with the vulnerability v and the efficacy eff according to the grid defined for that polygon. If there is more than one cruise ship position in one cell of the grid then the resulting efficacy can be obtained using the average or the minimum efficacy. The same rational applies to the vulnerability index. Since these maps are intended to represent a worst case scenario then the minimum efficacy should be considered.

Both indexes can be calculated for a specific location in time and they require different amounts of time to be calculated. The vulnerability index can be calculated very quickly because it only requires the system to perform distance calculations between positions in a map. The efficacy index is more elaborated because it requires an MMRO instance, which can take several minutes to build. The vulnerability index can be incorporated in a vessel monitoring system, such as the Oversee system [79], and this index can be automatically calculated when the user selects a cruise ship within the map. This functionallity was implemented in the prototype tool for demonstrations purposes and it is shown in Figure 58:

[31] *Surfacem* constructs a surface that represents a data grid with the vulnerability or efficacy values. A description of *surfacem* can be found in https://www.mathworks.com/help/map/ref/surfacem.html.

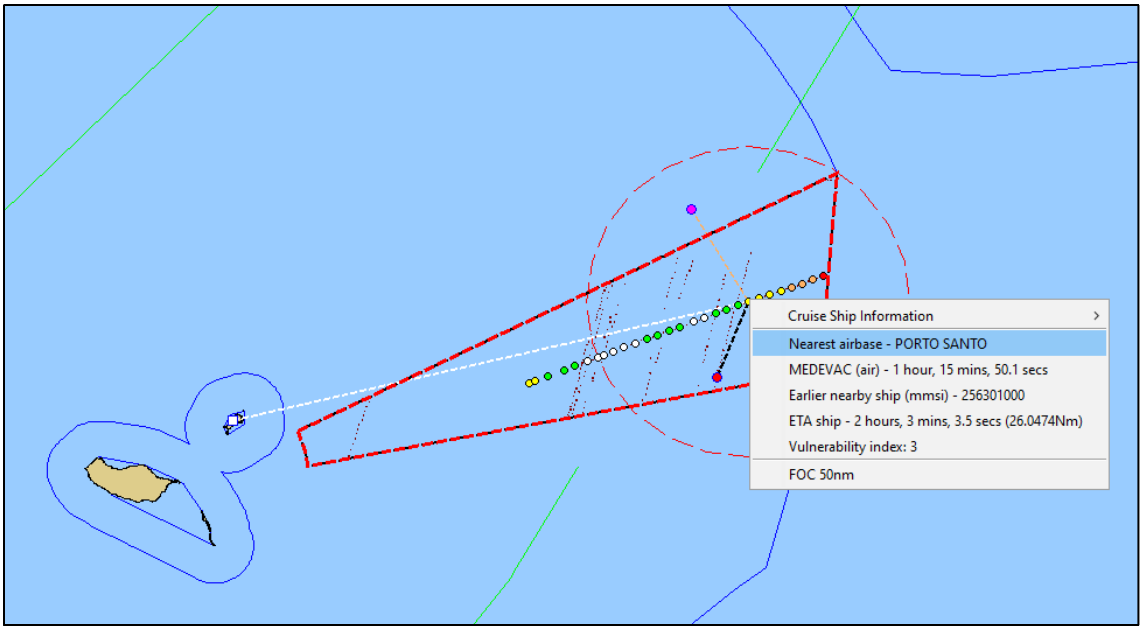

Figure 58. Vessel data: nearest airbase, closest opportunity ship and vulnerability index

Figure 58 shows the nearest airbase and the travel time required by the SAR helicopter in case of a MEDEVAC to reach the cruise ship Vision of the Seas (at a specific location and time). It also shows the nearest opportunity ship and the time it would need to reach the cruise ship location at its current speed. The vulnerability index described in formula (4.1), which is 3, is also presented.

The idea is to make this information available at each location of a cruise ship during its transit. To the best of our knowledge, these functionalities are not yet available in current maritime monitoring systems for SAR purposes.

4.4 Summary

This Chapter provides an overview of how the MMRO problem can be used to characterize a maritime area in terms of what could be the response to an incident that would require a MRO.

The first Section provides a simple formula to characterize areas remote from SAR facilities via the calculation of a vulnerability index. This formula was already tested in previous work but there is room for improvement. An alternative course of action to characterize areas remote from SAR facilities can be achieved by using the MMRO problem. This course of action implies answering to several questions: how many MMRO instances should one consider? How to set the MMRO instances and the survival times for the persons involved following an incident?. The second Section provides an example of how to build an MMRO instance based upon real data from a cruise ships and nearby opportunity ships. It is also presented a sensitivity analysis when the number of available helicopters is increased.

The third Section proposes a new approach to characterize areas remote from SAR facilities using an improved vulnerability index and the MMRO problem. To set the path for this approach, a procedure to characterize maritime areas regarding the efficacy of the SAR system response to an incident that requires an MRO is proposed. The efficacy is conditional to areas that are crossed by passenger cruise ships and the respective number of passengers. Since cruise ships are the vessels that have the largest number of passengers aboard it is crucial to identify the areas where these ships navigate and how many passengers are carried. Two scenarios are proposed to set the MMRO instances for each position of a cruise ship. Both scenarios do not consider any mortal victims prior to the alert which means that the potential number of lives to be rescued by the SAR system correspond to the total number of passengers and crew aboard. These scenarios pose the highest risk for the credibility and efficacy of the SAR system. For this reason the efficacy maps produced with the proposed procedure are presented as a valuable tool for monitoring cruise ships during their transits.

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Chapter 5

Results and Discussion

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5 Results and Discussion

This Chapter discusses the major findings related with the MMRO model as a tool to estimate the efficacy of the SAR system when responding to an incident that requires an MRO. The first Section discusses the potential of having information regarding the survivor's health and the impact on the rescue operation efficacy if a priority rule for rescuing them would take that information as input. The second Section discusses some of the challenges that would have to be met to incorporate the possibility of refuelling helicopters in to the heuristics. The third Section points the advantages and the limitations of the MMRO model as a method for estimating the efficacy of a SAR system response to an MRO. Finally, the last Section summarizes the most important aspects discussed in this Chapter.

5.1 Heuristics performance and the availability of survival times

Survival times are a critical piece of information to decision making with regard to the continuation or cessation of search and rescue activities. Predicting survival times for immersion victims is not a precise science and there is no formula to determine exactly how long someone will survive or how long a search should continue. The water temperature is the most common variable to determine the amount of time (in hours or minutes) a person has to survive if it remains immersed in the water for SAR planning purposes.

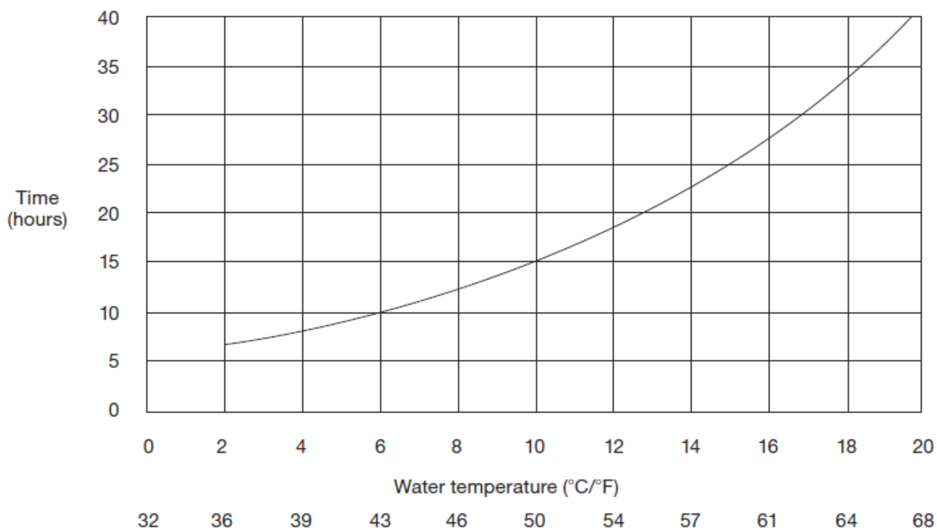

Figure 59. Survival time for people in the water wearing normal clothing

Source: [2, Fig. N-14]

Figure 59 shows realistic upper limit of survival time for people in the water wearing normal clothing, from time of entry into the water. The time curve is intended to be used in a decision making process by the SMC in order to produce an operationally feasible search plan that maximizes the probability of finding the distressed persons alive with the available search facilities. The first attempt to quantify the precise relationship between water temperature and survival time was made by George Molnar [210] in 1946 which was based upon an unspecified number of “selected” US Navy records of ship sinkings and aircraft ditchings during WWII. In 1962, Barnett [211] published an empirical predictive survival graph based on Molnar's original. The “Barnett” curve defines three areas for the life expectancy: “lethal” area, “marginal” area and “safe zone”. The large area between these two curves was labelled "marginal; 50% expectancy of unconsciousness which will probably result in drowning".

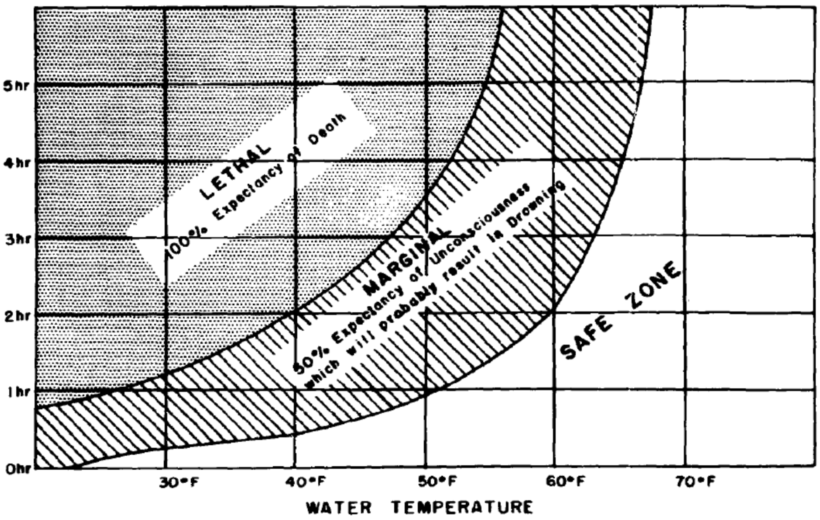

Figure 60. The "Barnett" curve: Time of life expectancy with no exposure suit

Source: [211]

Robertson and Simpson [212] present realistic survival times for a “standard man” immersed in the North sea in a variety of circunmstances, based on data from incidents available in the literature. A more comprehensive description in prediction of survival times can be found in the work of Tikuisis [213, 214].

In a real situation where there are persons immersed in the water, other factors will influence the survival time. The “cold shock”, inhalation of seawter, the rate of decline in core temperature, loss of will to survive and previous injuries are factors that will affect the survival time. For this reason, the functionality made available in the prototype tool to build MMRO instances does not force the survival time to be set or dependent of any of these factors. Instead, the prototype offers two options for setting the survival times:

the user sets the survival time for each search object individually or randomly generate these times with uniform distribution within the mission time window.

The question regarding survival times that this dissertation attempts to answer is: if the SMC knows, with a certain very high degree of certainty, the survival times of the persons involved in a maritime incident, what impact would this additional information would have in the co-ordination efforts of the rescue operation? What orientations or guidance the SMC would give to the SRUs that have to retrive persons from the water? If we add the assumption that the SMC also knows, with a very high degree of certainty, the drift of each person in the water over time and also the exact time each person would require to be retrieved from the water before (to live after being retrieved), then **the MMRO model would provide the optimal solution that would minimize the overall time the persons would have spent in the water or maximize the total number of rescued persons still alive** (depending on how the objective function is built). Assuming the SMC knows the survival times and the location of each person over time, the difficulty of obtaining an optimal solution would still remain on building the MMRO instance and solving it. Even if the exact location of the survivors drift is known, it would still be necessary to calculate arcs that represent feasible moves by vehicles between time-indexed positions (time index nodes in the graph). If an optimal solution is available then it would be sufficient for the SMC to communicate with each vehicle the sequence of survivors to be rescued to achieve the highest efficacy for the rescue operation.

If there is the possibility that the SMC can build the MMRO instance of a maritime incident (that requires an MRO) in a short time, but solving it optimally is not an option, then one can ask what a heuristic approach may offer in this situation. With this approach it is necessary to bear in mind that the proposed constructive heuristics described in Chapter 3 are intended to mimic a SRU rescue procedure when retrieving SAR objects from the water. There is no recommended procedure in the SAR doctrine for retrieving a large number of persons that are immersed in the water and scattered over a certain maritime area that guarantees a better efficacy. Guaranteeing safety for the crew's rescue ships and swiftness in the rescuing actions are the two major principles that should guide rescue facilities (specialized SRU or opportunity ships) in their rescue efforts[32]. When rescuing several scattered survivors by a rescuing vessel it is implicit that a priority rule is

[32] It may also be the case that rescuing ships do not have the necessary capabilities to hoist a person who may be suffering from hypothermia, especially after long-term immersion in water and especially when lifting them some distance such as to the deck of a high-sided vessel. For this reason the prototype tool allows the user to set the time required by a vessel to rescue a person, which can be different between rescue vehicles (nearby ships and helicopters) and it is not dependent on the type of vessel.

being followed by the vessel's captain. Usually, and following the swiftness principle, the captain chooses to rescue the nearest survivor and so on. This rule is similar to the distance criteria considered as a merit funtion to choose the vehicle/survivor assignment (see Section 3.3). The simple sequential constructive heuristic (see subsection 3.3.2) with a distance criteria (HC1d) fits very well in what would be the sequence of survivors to be rescued by multiple vehicles that are dispatched to the scene. The logic of this simple heuristic can easily be implemented. The SMC needs to know the nearest survivor to each vehicle in order to communicate to each vehicle the survivor they should be collecting. This information would have to be provided by a DSS able to maintain an updated maritime picture and also good communications between the SMC (coordinating operations in the MRCC) and the rescue units. The greedy sequential constructive heuristic with distance criteria (HC2d) or with ETA criteria (HC2e) would also require a DSS with adequate functionalities in order to provide the SMC the data he would need to coordinate the rescue.

With the assumption that the SMC knows the survival time of each person in the water and also their location over time, it should be reasonable to assume that this piece of information would bring an advantage in assigning vehicles to survivors. The computational tests (see Section 3.6) show that ETA and distance used in both types of constructive heuristics perform better when compared to profit for medium and larger instances (see Table 8). Using a more sophisticated heuristic such as the pilot method (full pilot method variants) also brings some surprise because using distance and ETA variants of HC1 and HC2 as subheuristics achieves better results when compared to using profit (see Table 9, Table 12 and Figure 40).

Unless the SMC is capable of obtaining the optimal solution in adequate time, these results suggests that knowing the survival times for the persons in the water may prove to be counterproductive. Using distance or ETA as a criteria to coordinate vehicles in their rescue activities seems to provide better results. This seems to be true if there is no other heuristic procedure that uses profit and outperforms the heuristic procedures that use distance and ETA.

5.2 Addressing the refuelling issue heuristically

Helicopters are an extremely valuable and versatile asset in any maritime SAR incident. However, like any mechanical device – and specifically one that flies in generally poor weather – it has its limits and as such must be managed and used with this in mind. Range and the maximum weight to perform stationary flight are two of the major factors for rescuing persons who are in the water. The helicopter's payload greatly depends on the amount of fuel it is carrying and the on scene weather conditions. For the crew it is a constant trade-off between fuel, payload and, most importantly, having a good power margin, especially for rescue operations. These factors determine the amount of time a helicopter can remain in a given area before leaving the scene.

The purpose to consider the possibility of refuelling a helicopter is the potential increase in range and subsequently the time it may remain in the scene. In November 2014, the MRCC Delgada coordinated a SAR operation [215] which involved the corvette Baptista de Andrade from the Portuguese Navy and the SAR Helicopter stationed at Lages airbase in Terceira island, Azores. The operation intended to rescue the skipper of a sailing vessel who had a serious head injury and was located at 950 nautical miles south of the island of Ponta Delgada in Azores. The corvette had to navigate south for 56 hours to reach the sailing vessel position and take the victim aboard. Then, it had to navigate 40 hours north to reach the operational range of the SAR helicopter in order to deliver the victim to an hospital. In Santa Maria SRR there are zones that take 4 days of navigation to reach. Most of the maritime areas within the Santa Maria SRR are outside the operational range of the SAR helicopter stationed in Terceira island.

There are two types of refuelling techniques for helicopters: air-refuelling or helicopter in-flight refuelling (HIFR). Figure 61 shows a helicopter being refuelled in-flight by the British Royal Fleet Auxilliary (RFA) Argus.

Figure 61. A Seaking Helicopter is refuelled in-flight from the deck of RFA Argus
Source: Royal Navy/MoD

The air refuelling technique requires the helicopter to be equipped with a long “probe” that fits the “drogue” of the air tanker. This equipment is not available for most of the helicopters who perform maritime SAR operations, such has the helicopters of the

Portuguese Air Force. In contrast, refuelling while hovering requires small adaptations to the fuel tank access from inside the helicopter's frame, which are simpler to perform and much more cost-effective. For this reason it is interesting to investigate the possibility of refuelling helicopters using the helicopter in-flight refuelling (HIFR) technique (see [216] for a video demonstration and [217, pp. 2–51] for safety procedures in HIFR operations). In this technique a helicopter receives an hoze from the ship's deck while hovering over it. Both helicopter and vessel are moving at a constant speed. The hose is connected to a ship fuel tank so that it can be pumped to the helicopter's fuel tank. This last technique is modelled in the MMRO problem by assuming that an extra range is given to an helicopter when it visits a replenishment ship node.

The next subsection provides examples of MMRO instances with the HIFR operation and discusses some features that make this type of operation difficult to conceive within a SAR operation.

5.2.1 MMRO instances with refuelling ship

The MDT prototype allows the planner to consider the trajectory of a vessel that is able to refuel a helicopter in the MMRO problem. The planner can set the trajectory of this vessel by choosing its initial position and several other positions on the map, assuming a constant speed between them for the necessary time calculations. For example, if the mission time windows is set for 10 hours, and the planner set the initial position of the refuelling vessel and the last intended position near the mass rescue scene, the system will automatically calculate all intermediate positions given the specified time-step.

There are two situations in the MMRO model where the HFIR operation component makes the MMRO solution to be unrealistic or impossible to be implemented in practice:

1) The MMRO solution with a HFIR operation is realistic for the helicopter in a way that the solution copes with the helicopter's flight plan, but there is an associated high risk, in which only in very special circumstances the flight plan (with the HFIR operation) would be accomplished in practice.
2) The MMRO solution is not feasible because it does not cope with basic rules and guidelines for a safe flight plan (for example, the MMRO solution implies that the helicopter travels a distance greater than its maximum range).

The first situation implies that there is a flight plan for the helicopter that complies with the MMRO solution, even if the flight plan is not a safe one and may present a high risk for the helicopter's crew.

Figure 62 shows the solution for a MMRO instance that fits the first situation. The instance has the following problem data:

- Mission start time (t_0): 2017-02-03 12:05:11.
- Four PIW located at 200 Nm west of Lisbon with survival times randomly generated with uniform distribution between the mission start time and eigth hours later.
- One SAR helicopter located at Montijo airbase, with 400 Nm of autonomy, recovery time of 5 minutes and average cruising speed of 100 Kts.
- No nearby vessels are dispatched to the scene.
- Mission duration: 8 hours.
- Time step: 5 minutes.
- One vessel capable of refuelling helicopter located 110 Nm west of cape Sardão[33] at the t_0 and starts ending north west to the PIW location.

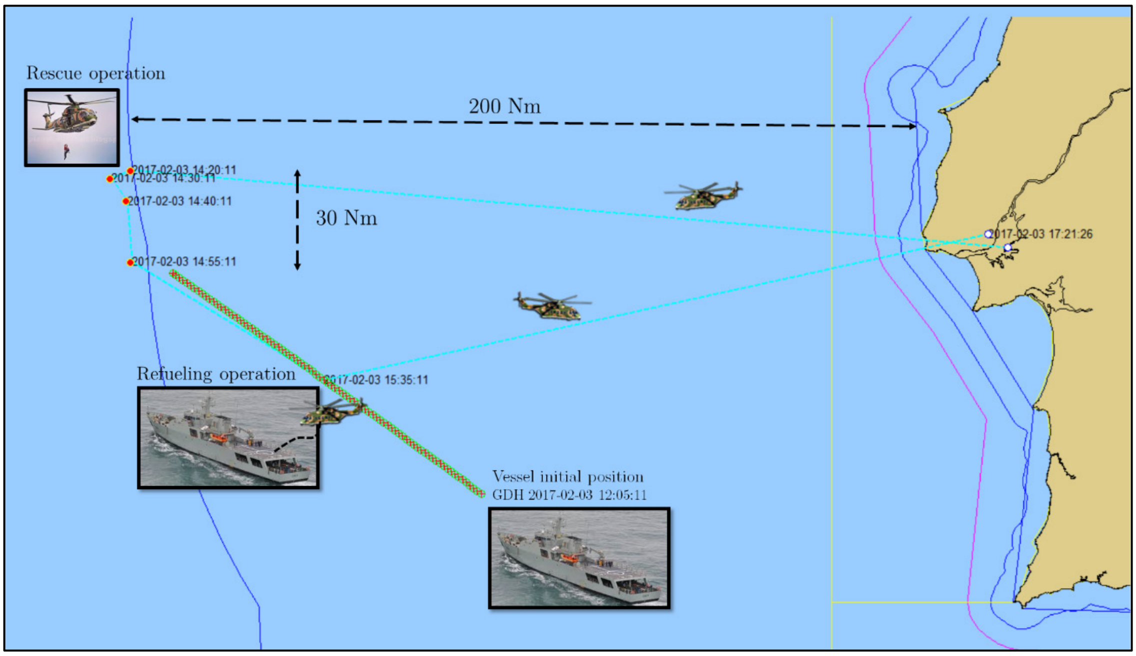

Figure 62. Solution for MMRO instance with vessel capable of refuelling

The solution (cyan dashed lines represents the helicopter's route) for the MMRO instance in Figure 62 could be accomplished with an adequate flight plan. Nonetheless, the presented solution involves a very high risk factor because if the refuelling operation is not achievable for some reason (for example, weather conditions or equipment fault) the most probable scenario for the helicopter would be to ditch in the water. In the absence of landing platforms, either a ship with an adequate landing platform or a rock in the middle

[33] Cape Sardão is a cape located in the Odemira municipality in Beja District in the Portuguese region of Alentejo.

of the ocean (assuming the helicopter can land there), the mission could be easily turned down by the helicopter's commander for lack of safety (regarding the crew's safety since the total flight distance is greater then 400 Nm). In [218] several guidelines are given relative to procedures endorsed by several nations regarding safety aspects in HIFR operations. One of these guidances [218, pp. 2A – ARG – 9] advise that HIFR operations should be conducted with sufficient fuel remaining for diversion to the nearest land base or carrier.

Figure 63 shows the optimal solution in the MMRO graph:

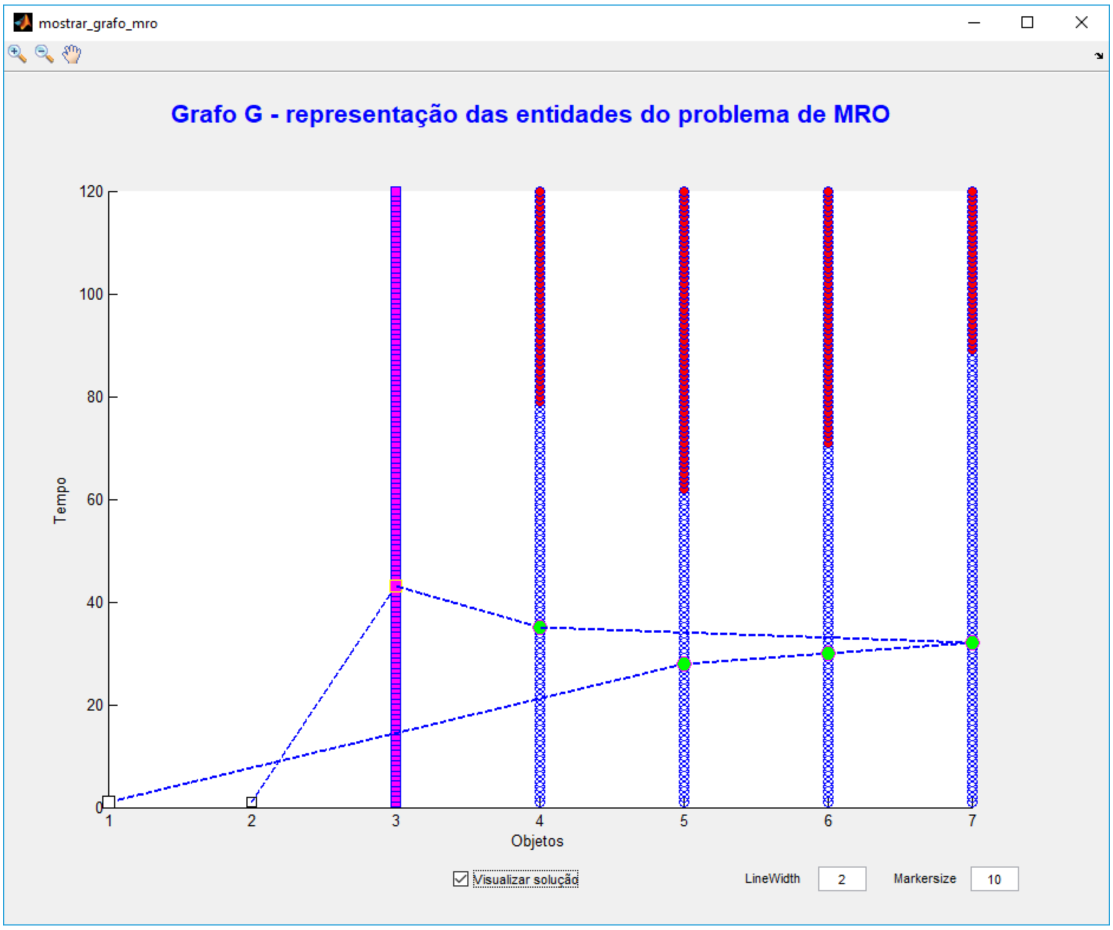

Figure 63. MMRO graph model with the optimal solution. Object 3 is the vessel with refuelling capability

Object 1 and 2 represent the Montijo and Figo Maduro airbases, respectively. The nodes corresponding to object 3 represent the location of the vessel from the initial time instance t_0 to the instant t_{120} (corresponding to the instance after the eight hour period).

Helicopter refuelling is only relevant in the MMRO problem if it is a necessary condition to retrieve someone from the water. Otherwise, it brings no benefit to do it at all. Technical issues and procedure safety norms can make the refuelling operation an acceptable option only in extreme and special circumstances. Refuelling before rescuing a person immersed in the water is only realistic if the extra weight from the fuel does not compromise the performance of the helicopter for stationary flight and the helicopter is able to return home safely.

The second situation is related to the helicopter's autonomy constraint within the MMRO model. The inequality 3.15 (see Section 3.2) does not guarantee that the helicopter cannot travel a distance larger then its maximum range if it is refuelled. Figure 64 shows an MMRO instance where the helicopter travels a distance larger than its autonomy before being replenished:

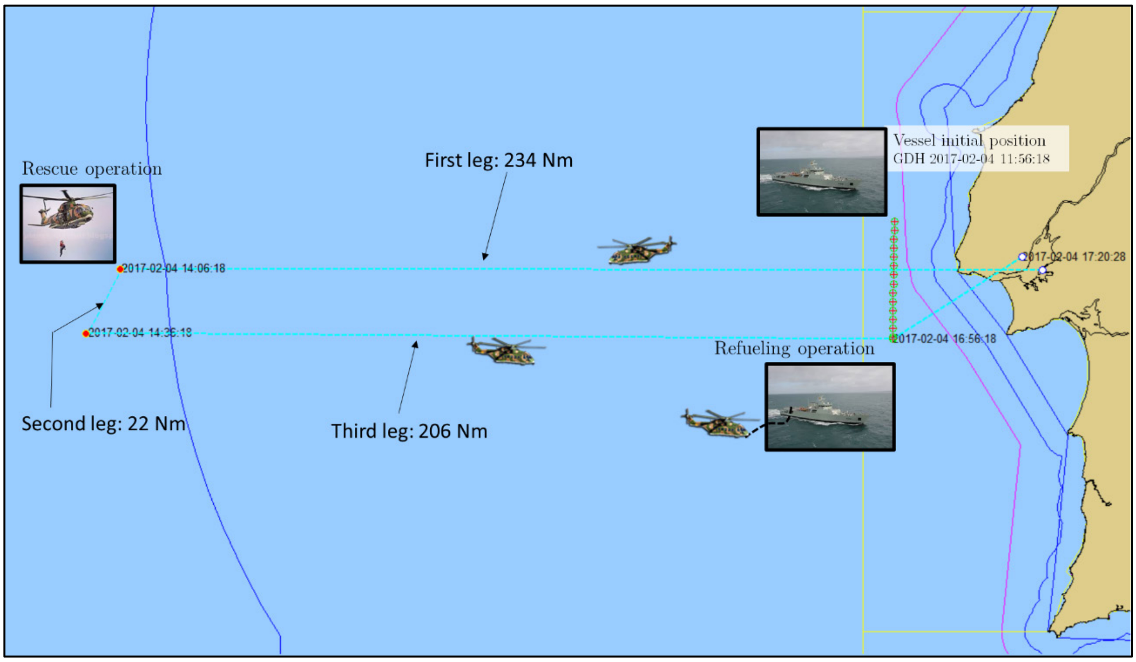

Figure 64. Unpractible MMRO solution with vessel capable of refuelling

The solution depicted in Figure 64 is the optimal solution for the MMRO instance, but it is not compatible with a feasible helicopter flight plan due to the autonomy violation. In this example, the sum of the first, second and third legs total 462 Nm before the replenishment operation. The helicopter would ditch into the water before it reached the vessel for refuelling. This particular case of autonomy violation can be easily observed within the helicopter's autonomy constraint in the vehicle flow formulation because the sum of all visited arcs must not exceed the autonomy parameter independently of their sequence.

5.2.2 Vehicle/survivor assignments with refuelling

In this subsection we discuss the necessary changes to the constructive heuristics presented in Section 3.3 to consider the possibility of HIFR for helicopters.

It is possible to develop procedures within a heuristic framework where it is easier to guarantee the feasibility of the solution being built. For example, the autonomy problem presented in the previous subsection can be controlled with a state variable that keeps the distance travelled by a helicopter from the last instant that it visited an airbase or a vessel to be refuelled. The constructive heuristics proposed in Chapter 3 use the concept of vehicle/survivor assignment to build the routes for the available rescue vehicles during the construction of the partial solution. This concept requires the calculation of several state parameters that are used to evaluate if a certain vehicle can, in fact, retrieve a certain SAR object from the water. The same logic can be applied to the possibility of refuelling a helicopter after retrieving survivors from the water. It is important to note that refuelling a helicopter before rescuing any survivors implies (assuming the start and end depot are the same) that it has enough fuel to return to base. It also presents the risk of taking extra weight that makes stationary flight impossible. For this reason we only consider the possibility of refuelling when the helicopter is returning to base. It may also happen that a helicopter may still rescue more survivors before being refuelled and return to base.

The HFIR operation must be considered in two specific steps within the constructive heuristic framework:

- During the assessment of a **feasible vehicle/survivor assignment** (step 4 in algorithm 3.2 and step 4 and step 15 in algorithm 3.3). In this case, the **vehicle/survivor assignment** between a helicopter k and a survivor s must satisfy the following conditions:
    - Survivor's weight does not exceeds the remaining capacity of helicopter k.
    - Helicopter k must be able to perform HIFR after recovering a survivor and return to base within the mission time window.
- During the step where the helicopter is examined to terminate its route and return to base (step 13 in algorithm 3.2 and step 21 in algorithm 3.3).

It is important to note that the procedure should be used to assess if a certain helicopter/survivor assignment is feasible or not independently of having to refuel afterwards and return to base. If the helicopter/survivor assignment is feasible then the survivor index is added to the helicopter vector vsk as well as the survivor node index to the vector vss. In future iterations, it is necessary to evaluate the next possible nodes that the helicopter may visit. If the helicopter cannot rescue any more survivors, then the step where the heuristic examines the helicopter to end its tour must contemplate the possibility of refuelling before returning to base.

Figure 65 presents a diagram with the conditions that have to be verified to assess if a certain helicopter/survivor assignment is feasible with the possibility of refuelling.

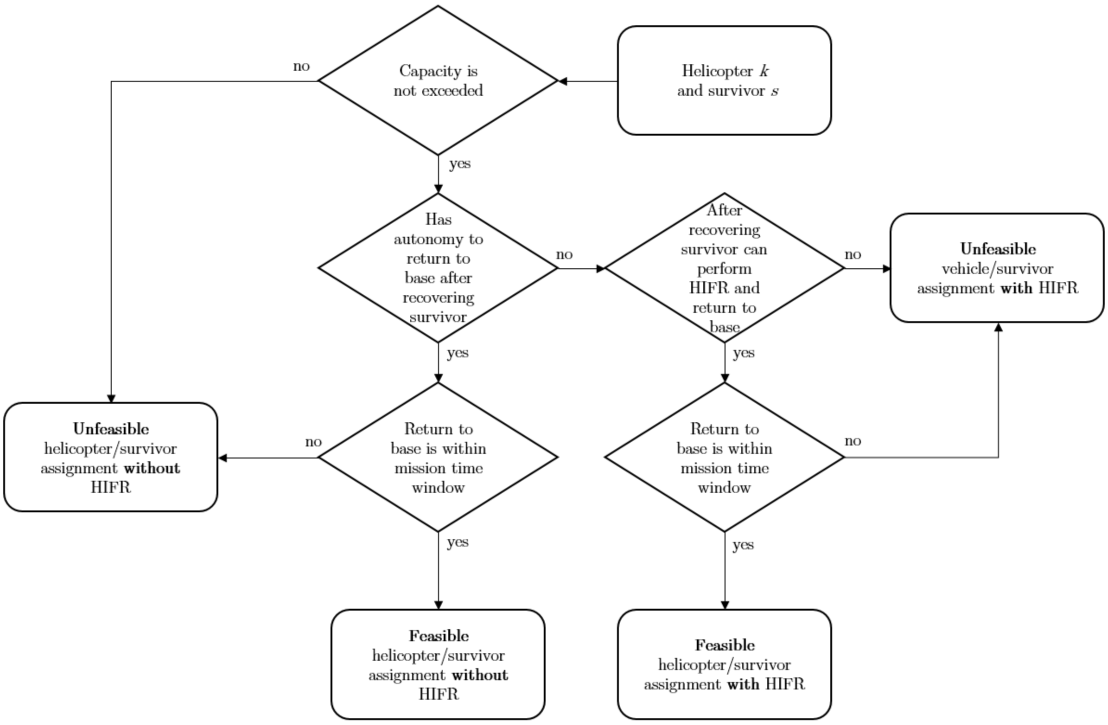

Figure 65. Procedure to check helicopter/survivor assignment feasibility with HIFR

The procedure for determining if a helicopter k can rescue a survivor s and perform a refuelling operation afterwards implies checking a sequence of conditions regarding the position of the helicopter k and its status (current capacity, distance travelled) and also the possible next nodes to be visited. The first condition to be verified is if the remaining capacity of the helicopter k is enough to receive survivor s. The next condition to be verified is if the helicopter cannot rescue the survivor s and return to base immediately afterwards. If true, then it is necessary to evaluate the possibility of refuelling (after rescuing survivor s) and return to base. If refuelling is possible and it is also possbile to return to base then the helicopter can rescue the survivor s. In this case, if the assignment (k, s) has the highest merit, then (k, s) is added to the solution. It may still be possible for vehicle k to keep rescuing survivors before refuelling and ending its tour.

The difficulty in permitting the HIFR operation into the MMRO model stands with the complexity of managing the helicopter power margin during its tour. The helicopter needs an adequate power margin to perform stationary flight (for rescuing a survivor) and also to perform HIFR and this changes with the weight and the amount of fuel at each moment. This power margin depends on several factors, including the weight (fuel, crew, survivors, equipments onboard), engine power, number of engines, altitude and wind speed. Most routing problems with helicopters address the problem of transportation of personnel between oil platforms (see [219–223]) and do not require stationary flight to pickup a passenger (because a landing platform is available) and thus the power performance is not a requirement.

Although it is easier to control the distance travelled by the helicopter heuristically, other circumstances require attention if we want the MMRO solution to be realistic and the HIFR issue justifies a thorough investigation to incorporate it on a mixed integer programming model or within an heuristic framework.

5.3 MMRO model advantages and limitations

In this Section we present the advantages and limitations of the MMRO model as a tool to assess the SAR efficacy to an incident that requires an MRO.

5.3.1 Advantages

The MMRO model takes into consideration the three major criteria for a maritime area to be considered "remote from SAR facilities": number of persons at risk, recovery capacity of nearby ships and SRU, and survival times. To the best of our knowledge, no other similar models consider these parameters in a integer programming model to estimate the efficacy of a SAR operation. The MMRO model also considers the maritime drift for different types of objects, which affects the time these are recovered and ultimately the health condition of the survivors. The discretization in time of the location of each object simplifies the construction of a time-dependent objective function that relates time and the profit for retrieving a certain object from the water.

Time related statistics (drift time of objects, duration of the rescue operation, time spent in the water by objects, etc) can be estimated and these are paramount to an effective assessment of SAR capabilities by the authorities. The speed of SRUs and SAR facilities (nearby ships) are considered and these affect the duration of the rescue operation and the time each object spends in the water.

An interesting advantage of the MMRO model is the capability to model different incidents that require an MRO that occur at different locations but at the same time. The possibility of multiple occurences should not be underestimated and the terrorist threat can be a plausible cause behind multiple incidents.

Overall, the MMRO model can be used to assess the benefit of having additional facilities, such as helicopters, in case of an incident that requires a MRO. This assessment is crucial when authorities need to present arguments that support strategic decisions regarding equipment acquisition. The efficacy estimation can also be used to characterize a maritime area that is navigated by large passenger cruise ships. Knowing which maritime areas present a higher risk of the SAR system not being able to respond effectively to an incident is of maximum importance and the MMRO model can provide that answer.

5.3.2 Limitations

Concerning the purpose of assessing the SAR efficacy using the MMRO model, there are two types of limitations. The first has to do with the difficulty to predict the exact consequences or effects of future maritime accidents that requires an MRO. The consequences or effects of an incident have a huge impact *per si* on the overall MRO efficacy. The swiftness of the SAR response (rescue operation) determines the overall efficacy only to a level consented by the consequences of the incident. The next two considerations resume the first type of the MMRO limitations:

i. Predicting the drift of objects in ocean areas with high accuracy is a very difficult endeavour, even for today's computational capability and sophisticated drift models. Current techniques for estimating the drift of a SAR object are used to calculate search areas that guarantee a high probability that the SAR object is within that area in a specific moment in time. The MMRO model requires the exact location of each object during the mission time window.

ii. It is almost impossible to determine the exact effects from a maritime incident in order to assess the SAR capability to respond to it. All the scenarios considered in the present work are pure hypothetical. In order to assess the SAR capability, the scenarios to be evaluated should be designed by experts within the SAR agencies and other related entities (cruise ship operators, port authorities).

The second limitation has to do with the MMRO model ability to represent the major features of the rescue operation involving a complex dynamic between several SAR vehicles and a huge number of drifting objects.

iii. The "rescue" term has a broad meaning that involves three activities: "retrieving people in distress; attending to their immediate needs; and transferring them to a place of safety". The MMRO model only deals with the retrieval of people in distress. An extended view of the MMRO problem can be considered in which the victims, after being rescued, are moved to an adequate facility (hospital).

iv. The MMRO model assumes that the location of the objects is known throughout the mission duration. This assumption takes out the "search" component from the

Search and Rescue. This assumption may be closer to reality than it appears at a first glance. In a maritime incident, fixed wing aircraft can be sent to the incident's location and track all the objects in the water, keeping an updated sea picture for the SRUs that will perform the recovery of the survivors. Nonetheless, this "updated picture" depends largely on the local weather conditions and if the rescue is performed during daylight.

v. The MMRO model only accepts two types of speeds depending on the type of SRU. For rotary wing SRU (helicopters), the model can consider different passenger capacities but it doesn't allow different speeds. The model considers, at the present moment, two types of vehicles, which are homogenous regarding speed (all the vehicles of the same type have the same speed) and this feature simplifies the construction of the layered graph. Each different value for the vehicle's speed would imply the construction of a new vehicle adjacency matrix for that specific speed. This would greatly increase the data structures within each MMRO instance.

vi. The time required to rescue an object is a parameter of the MMRO model which depends exclusively on the type of vehicle. It is not difficult to accept that taking someone out of the water with a helicopter won't take the exact same time if done several times. The rescue operation performed by helicopter which is hovering an alive PIW depends upon several factors: weather conditions, PIW state (if it is in panic or not), proficiency of the crew (especially the winchman).

vii. The helicopter performance for recovering survivors greatly depends on the amount of fuel it is carrying and the on scene weather conditions. For the crew it is a constant trade-off between fuel, payload and, most importantly, having a good power margin. This trade-off between fuel, payload and power it is simply not considered in the MMRO model.

viii. Survival times are a parameter of the model but the availability of this information in the rescue operation depends on the type of scenario being evaluated. In a scenario where we admit that available technology allows the SAR systems, and the SRU in the scene, to know the critical state of each survivor, then the survival times should be used in the heuristics to find a better rescue plan. Otherwise, the survival times are not used within the heuristics but are used for evaluating the efficacy of the operation.

ix. The time step is also a parameter of the MMRO model that greatly influences the size of the problem. Small values of the time step will provide greater detail on the time instances for which the objects are recovered, while larger values will tend to give inaccurate time instances.

x. The solution states that a vehicle has to recover all or none of the persons in each SAR object. If a liferaft has 30 persons aboard, the current model does not allow that a vehicle may recover one person and leave the remaining 29 on the liferaft. In real operations it is plausible that a helicopter may recover an injured survivor who is aboard a liferaft with more persons aboard and leave the rest afloat to be pick up later by another SRU.

5.4 Summary

This Chapter discusses the key features of the MMRO model and how it copes with the purpose of estimating the efficacy of the SAR response to an incident that requires an MRO. The availability of survival times is another important feature of the MMRO problem that has direct impact in the efficacy of the SAR system response.

In the first Section a distinction is made between the survival times the SAR system use for planning purposes and the real estimates of these values. The first ones are upper limits of the second and they are used to determine when a search operation should end. For the purpose of estimating the efficacy of the SAR system response, the MMRO model requires the real survival times and not their upper limits. With the necessary assumptions about survivors location through time, survivor's health condition (survival times) and building the MMRO instance promptly after the alert is received, the SMC does not have his task made easier if he is not capable of obtaining the optimal solution for the vehicles routes. The computational experiments with different variants of the constructive heuristics show that the priority rule based on profit does not outperform the priority rule based on distance and ETA. Even with a more sophisticated heuristic such as the pilot method, this relation in performance (concerning the pilot heuristic variants) verifies as well.

In the second Section we discuss the refuelling operation and how this can be included in to the heuristics presented in Chapter 3. Although the vehicle flow formulation for the MMRO model allows the vehicle to visit a node and gain some extra range, several issues require attention to guarantee that the heuristic solution is feasible in practice. The HIFR operation is considered an emergency operation and should only be conducted if no alternatives are available. What makes this type of operation desirable is the fact that it is fairly easy and economic[34] to mount the required facilities onboard a vessel to be able to refuel a helicopter without having to land on it. One of the issues that have to be considered to guarantee that the MMRO solution is realistic in practice is an adequate model for the power margin during the helicopter route. Without such a model, one cannot guarantee that the helicopter can perform stationary flight.

The third Section provides an overview of the advantages and limitations of the MMRO model as a tool to assess the efficacy of the SAR system response. The advantages of the MMRO model rely on the maritime drift and the time dependent objective function that is related to the survival time of the survivors. These are critical factors in a maritime SAR operation, since they are necessary to recreate an incident in ocean waters where assistance can take several hours to reach the scene. During this time, it may happen that survivors, whether they are PIW or persons inside liferafts or lifeboats, will disperse due to weather conditions. On the other side, the MMRO model presents several limitations: these are related with the difficulty to predict the effects of a maritime incident that requires an MRO and the other is related with the completeness of the model. Althought there are more limitations than advantages, the “limitations” part opens several paths for further investigation.

[34] The economic factor in preparing a vessel with the equipments necessary to perform a HIFR operation is smaller when compared to other options such as acquiring helicopters with increased maximum range or acquiring a landing platform dock (LPD) capable of carrying heavy helicopters.

Chapter 6

Conclusions

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6 Conclusion and Future Work

This Chapter concludes the dissertation. It begins with a Section where the most relevant points from the preceding Chapters are summarized. This is followed by some concluding remarks that put those results in perspective and by comments on future work.

6.1 Summary and conclusions

In this dissertation, we propose a new combinatorial model to address the problem of estimating the efficacy of the SAR system response to a maritime incident that requires a mass rescue operation (MRO). This problem is designated by Maritime Mass Rescue Operation problem and belongs to the family of Combinatorial Optimization Problems.

To address the MMRO problem we discuss a binary linear programming formulation that is a profit variant of a generalized vehicle routing problem. This formulation consists in a vehicle flow model formulation based on a huge layered graph that represents the drifting trajectory of objects in time. The vehicle flow model considers a heterogeneous fleet of vehicles that are required to retrieve a set of SAR objects that are drifting in the water. Assuming that all simulated data (object's drift, survival times, recovery times) meet the terms of a real maritime incident, the compliance of the model depends significantly on the time step parameter. If too large, the solution's routes won't match the times required by the vehicles to move between locations and the vehicle's travel and retrieving times can't be used to assess the operation's efficacy. If too small, there will be variables in the solution that bring no additional or useful information and the size of the instances may be too time-consuming to build.

Two types of constructive heuristics were proposed to obtain a feasible solution to the problem. We also implemented a pilot method based upon several different variations of the constructive heuristics acting as sub heuristic (or pilot heuristic). The availability of the survival times embedded in the constructive heuristics and pilot method was analyzed and compared with the distance and ETA. When the objective function is a function of the survival times and these are a time dependent function, results show that the pilot method using a greedy pilot heuristic with ETA or distance as merit function provide better results than using profit. These results provide evidence that support the idea that if the SMC has full knowledge of the survival times as well of the exact location of each SAR object over time then, unless he can obtain the optimal MMRO solution, this information will not make is coordination task easier.

A pilot method was also implemented to provide better quality solutions. To the best of our knowledge, this is the first application of this type of method to solve vehicle routing problems. Different variants were tested based on the sub heuristic and the sequence length of feasible assignments that are added to the master solution. The sequence length and the number of assignments in each level (position) of the sequence were evaluated. Our results showed that the partial pilot method with two levels provided better results than its counterpart with only one level. The challenge in tunning the pilot method stands in finding a compromise between sequence length and the number of assignments to be evaluated in each level. The idea is to find an acceptable time demanding pilot method with adequate solution quality. But this relation may be dependent on the problem data. More computational experiments are required to further investigate this subject.

The MMRO model was used to build instances based on real data regarding the location of a cruise ship during its transit between Funchal (Madeira Island) and Malaga (Spain) in April 2016. The consequences of the incident were designed and grouped into scenario variants in order to assess the efficacy of the SAR response using nearby vessels and the SAR helicopter stationed at Porto Santo airbase. Results show that the area where the incident was located can be perceived as an area remote from SAR facilities. This is due to the fact that there are time periods where the shipping density is low and the nearest vessels may not reach the incident in adequate time. The scenario variants show that the SAR efficacy, with a "conventional" response, is largely dependent on the survival times. The MMRO was also used to assess the effect on the efficacy if more helicopters were available. Whether these scenarios are adequate to analyze strategic solutions regarding the acquisition or the enhancement of current capabilities (maintaining two helicopter crews instead of one at Porto Santo) depends on the strategic view of the authorities and operational objectives for the SAR capability.

One of the questions that this dissertation tried to answer was if the availability of survival times by the SMC would influence the efficacy of a rescue operation. It is important to note that if the SAR system has full knowledge of the drift of each SAR object and the respective survival times, then the optimal solution of MMRO problem provides a rescue plan with maximum efficacy. To answer this question several heuristics procedures were developed based on criteria that reproduce the standard procedure adopted by the SMC when coordinating SRUs in a rescue operation. These standard procedures aim to rescue persons as swiftly as possible and we propose the use of merit functions based on distance and ETA between a vehicle and a survivor to decide which assignment should be made. Our results show that the merit functions based on distance and ETA outperform the merit function based on profit (profit is a function of the survival times used as the objective function). This implies that without the optimal solution for the MMRO problem, knowing the survival times of the persons that require assistance may not prove to be beneficial to the efficacy of the rescue operations.

6.2 Future work

The MMRO model presented in this dissertation is intended to serve as a tool to assess the capability of a SAR system to cope with maritime mass rescue operations. SAR doctrine states that survival times, number of survivors, weather conditions and SAR facilities availability are among the most important factors that will determine the efficacy of a rescue operation. The problem of assessing the SAR capability remains a challenge for Coastal States such as Portugal were the investment in SAR programs have to face competition with other governmental programmes. But economic factors, such as the cruise ship industry and its contribution to the economy in many Coastal States has called for attention to guaranteeing safety in coastal, but also ocean areas within the Coastal's State SRR. This is why it is important to study methods for assessing the SAR capability to cope with maritime incidents that require MRO. Keeping this purpose in mind, future research topics can be grouped into three categories:

- Improving the MMRO model as a tool to assess the maritime SAR capability to respond to incidents that require a MRO.
- Improved heuristics for the MMRO problem.
- New research methods for identifying areas remote from SAR facilities.

As for enhancing the MMRO model, several issues require further investigation:

i. Generalize the vehicle flow formulation for vehicles with different velocities. In the current MMRO model, each velocity implies the construction of an adjacency matrix that defines the feasible arcs between time-indexed nodes. Currently, there are only two available velocity values (one for nearby ships and another for helicopters). For example, we may need to consider two types of helicopters that travel with different cruise speeds. This would implie additional data structures which can increase the time for building a MMRO instance. Another interesting challenge would be to consider different velocities depending on the distance between time-indexed nodes.

ii. Allow multiple visits to a cluster. The MMRO model considers each object as a cluster of nodes that represent the position of that object in time. In the vehicle flow formulation each cluster can only be visited once. This implies that a liferaft with 50 persons aboard can only be retrieved by a vessel, since a helicopter can hardly carry that amount of passengers. It would be interesting to study new formulations where each cluster can be visited more than once and each visit could carry a variable amount of persons.

iii. The MMRO model only assumes one survival time associated to a liferaft or life boat. This assumption is not realistic because it is likely to have injured persons aboard a liferaft. Considering different survival times for each person in a liferaft or lifeboat would greatly improve the flexibility of the MMRO model.

iv. One of the advantages in discretizing time is the possibility of starting the vehicles tour at different moments in time. This possibility is more realistic because if several helicopters are dispatched to the scene it is likely they do initiate their tour from different locations and may have different readiness times. An interesting feature to add to the MMRO model would be to define different starting moments for the available vehicles.

v. The refuelling issue is “a must have” feature in the MMRO model because of the exceptional circumstances of a MRO. Due to the large number of lives at risk it is conceivable that emergency operations like HIFR can be performed. Additionally it is also interesting to consider the option of landing platform type vessels to act as temporary airbases during the rescue operation.

vi. Following the previous research sugestion, it is interesting to consider several helicopter sorties in the MMRO model. Several integer linear programming models have been proposed for helicopter vehicle routing problems concerning the transportation of personnel between oil platforms. The MMRO model would benefit of such feature.

vii. One of the difficulties in using the MMRO model is the time required to build the necessary data structures, especially the vehicle adjacency matrixes that verify the feasibility constraints for moving between time-indexed nodes. It would be interesting to investigate ways to reduce the calculation time of these data structures.

There are several available heuristics methods that can be investigated to solve the MMRO problem. The pilot method variants that were tested represent only a very small set of possible variants that can be used to solve the problem. Little is still known about the suitability of this type of heuristic in solving combinatorial optimization problems. Regarding the MMRO problem, more computational experiments are required. It is not clear if the performance of the constructive heuristics or the pilot method variants depends on the location of the survivors (their initial dispersion) and their survival times. It would be interesting to solve the MMRO problem with other different metaheuristic such as genetic algorithms, tabu search, simulated annealing or large neighborhood search (LNS).

The efficacy of the SAR system to respond to an incident that requires an MRO is related to the concept of "area remote from SAR facilities". The IMO Maritime Safety Committee [195] recomends Coastal States to identify these areas within their own SRR as it is recognized that incidents in these areas may be difficult to provide the required assistance in due time. Although the recommendations state the factors which may make an area remote from SAR services, they do not tell how to use these factors and classify these areas. In [105] is proposed an algorithm to classify maritime areas regarding cruise ships but more research is needed since the classification requires the measure of the risk associated to a type of vessel and it's respective activity. It is important to distinguish the risk associated to passenger cruise ships from the risk associated to the transportation of liquid gas by large tankers or any type of vessel engaged in a specific economic activity. Investigating risk measures to classify areas remote from SAR facilities is a critical research topic that must be pursued.

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Annex A - List of SAR Objects and leeway values

Table 26 shows the parameters for obtaining the leeway speed. For example, the leeway equation for a basic raft without drouge is $0{,}07U + 0{,}04$, where U represents the wind velocity.

Table 26. Leeway parameters of obtaining leeway speed.

SAR object	Leeway parameter	
	Multiplier	**Modifier**
Light displacement cabin cruisers, without drogue	0,07	0,04
Large cabin cruisers	0,05	0
Light displacement cabin cruisers, with drogue	0,05	-0,12
Medium displacement sailboats, fishing vessels	0,04	0
Heavy displacement deep draft sailing vessels	0,03	0
Surfboards	0,02	0
Basic raft without drogue	0,07	0,04
Basic raft with drogue	0,05	-0,12
Raft with canopy and without drogue	0,084	0,048
Raft with canopy and drogue	0,06	-0,144
Raft with ballast pockets and without drogue	0,056	0,032
Raft with ballast pockets and with drogue	0,04	-0,096
Raft with canopy, ballast and without drogue	0,07	0,04
Raft with canopy, ballast and with drogue	0,05	-0,12
Raft deep ballast without drogue	0,03	0
Raft deep ballast with drogue	0	0
Canopy and deep ballast, without drogue	0,05	-0,12
Canopy and deep ballast, with drogue	0	0
Circular raft with canopy, symmetrical ballast without drogue	0,05	-0,12
Circular raft with canopy, symmetrical ballast with drogue	0	0
EPIRB	0	0
PIW (vertical)	0,005	0,07
PIW (Sitting)	0,012	0
PIW (Horizontal - Survival Suit)	0,014	0,1
PIW (Horizontal - Scuba Suit)	0,007	0,08
PIW (Horizontal - Deceased)	0,015	0,08

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Annex B - Criteria for determining “areas remote from SAR facilities”

The following criteria are considered relevant in determining what constitutes an area remote from SAR facilities:

1. the number of people at risk;
2. the nature of the risk and whether containment strategies can mitigate its effects, in particular whether the effects of the incident can be so contained as to enable those at risk to remain on board until rescued, or for a period prior to eventual evacuation, thus extending the time to recover;
3. the availability of SAR facilities and other resources which may be deployed in order to contain the incident and keep those at risk on board until rescued, or for a period prior to eventual evacuation, thus extending the time to recover;
4. the total recovery capacity of SAR facilities available to reach the scene and recover those who have taken to survival craft within the five day “time to recover” parameter and/or within survival times;
5. .any shortfall between the number to be recovered and the capacity of those SAR facilities available;
6. the distance (in time) between individual SAR facilities’ start points and the scene of the emergency;
7. the prevailing sea conditions, both on scene and encountered by SAR facilities proceeding;
8. the prevailing weather conditions, both on scene and encountered by SAR facilities proceeding;
9. any restrictions on SAR facility deployment which limit or remove their ability to respond even if theoretically within reach of the scene of the emergency;
10. the ability of those at risk to survive in the prevailing weather and sea conditions until they can be recovered (that is, for a maximum of five days according to the“time to recover” parameter);
11. the ability of available SAR facilities to recover those at risk in the prevailing weather and sea conditions;
12. any shortfall between the time taken to recover those at risk and the five day“time to recover” parameter and/or survival times in the prevailing conditions;
13. availability and quality of communications; and
14. effective co-ordination of search and rescue response.

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Annex C - Vulnerability maps for cruise ships

The following figures show the vulnerability maps produced by Nascimento [105] for several maritime areas within the Portuguese SRR. The AIS data used to calculate the ETA of nearby ships to each position of the cruise ships during their transits is set between january 1 and december 31 of 2016. The resulting map is built using an histogram of the maximum of the indexes calculated for each square.

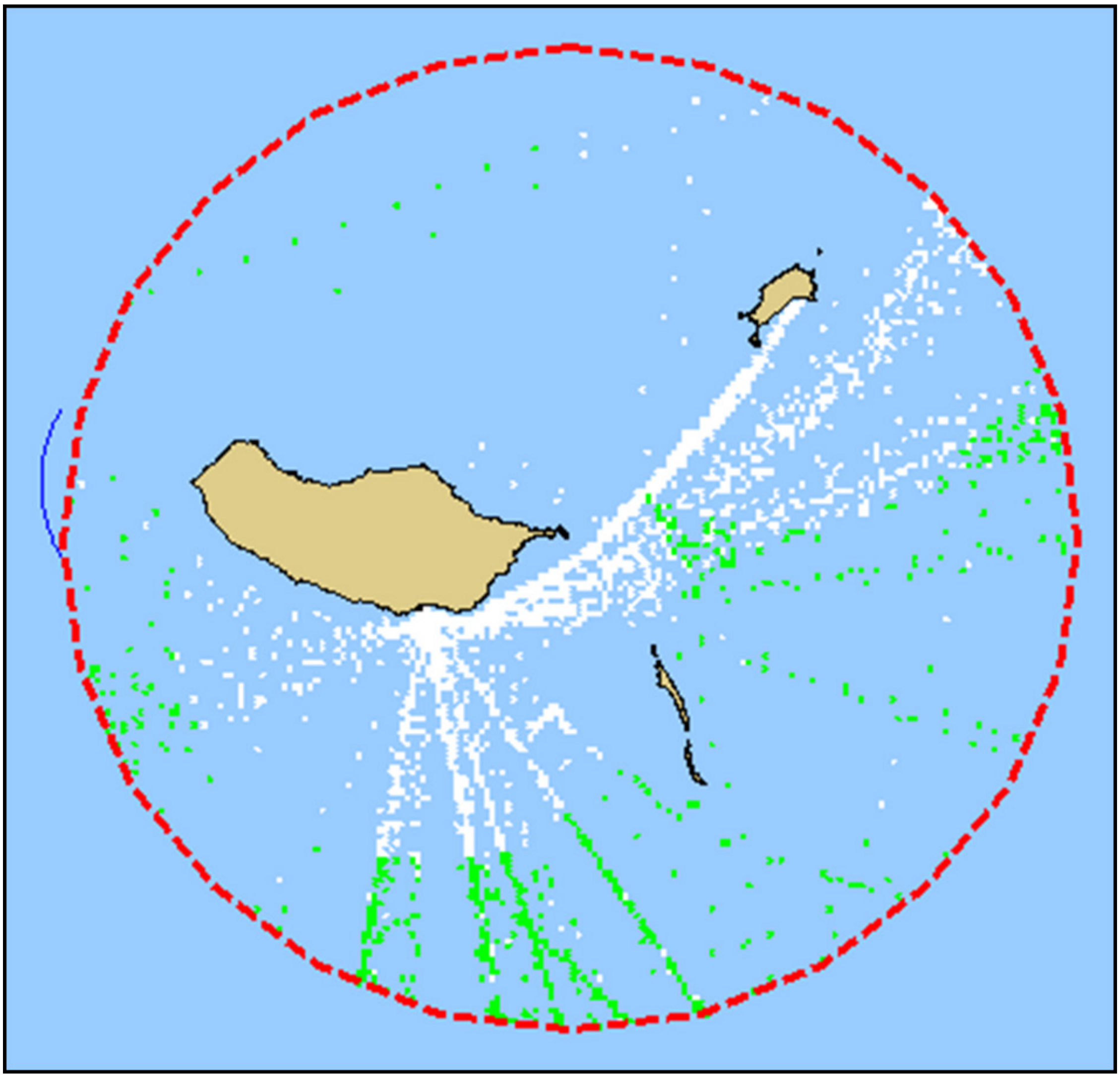

Figure 66. Polygon: “Madeira centro”. Mesh: 1 Nm^2

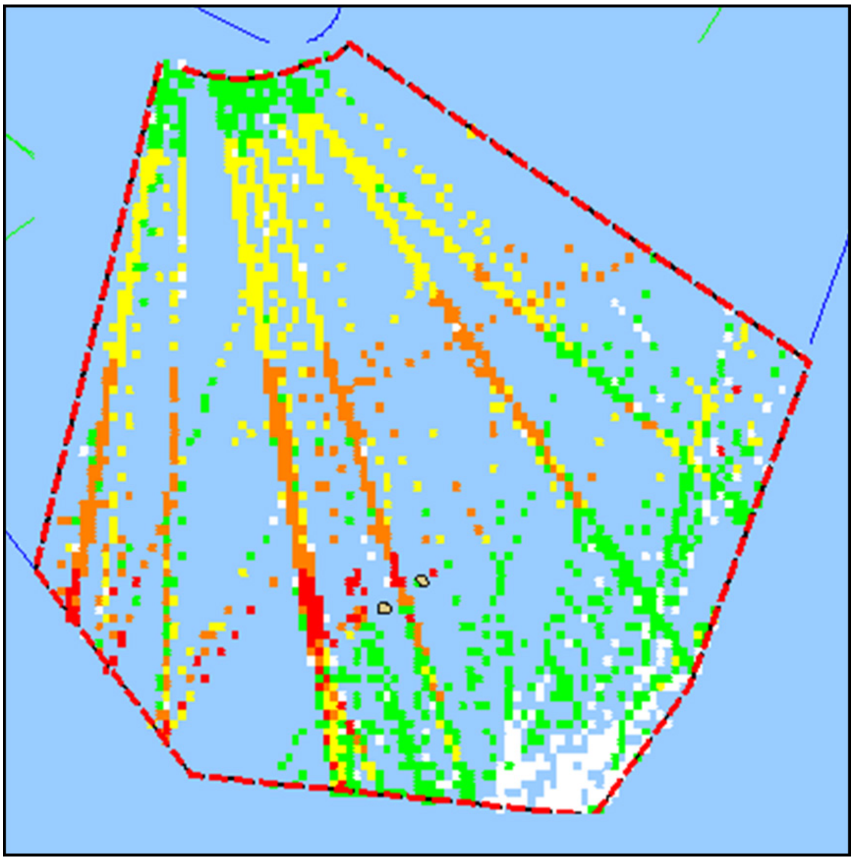

Figure 67. Polygon: "Madeira sul". Mesh: 2 Nm^2

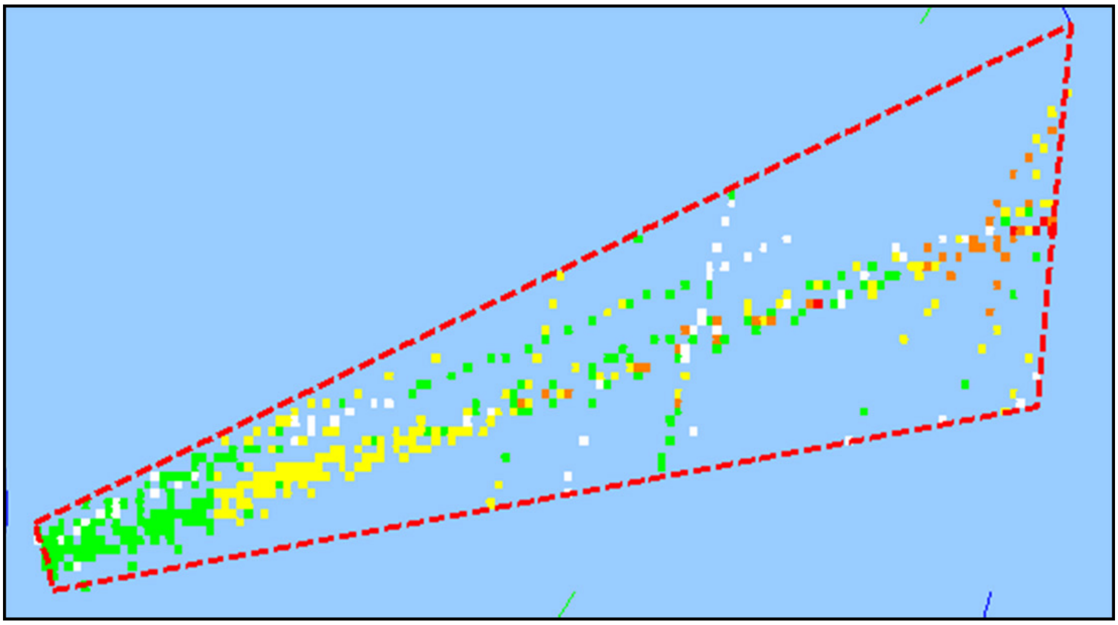

Figure 68. Polygon: "Madeira este". Mesh: 1.5 Nm^2

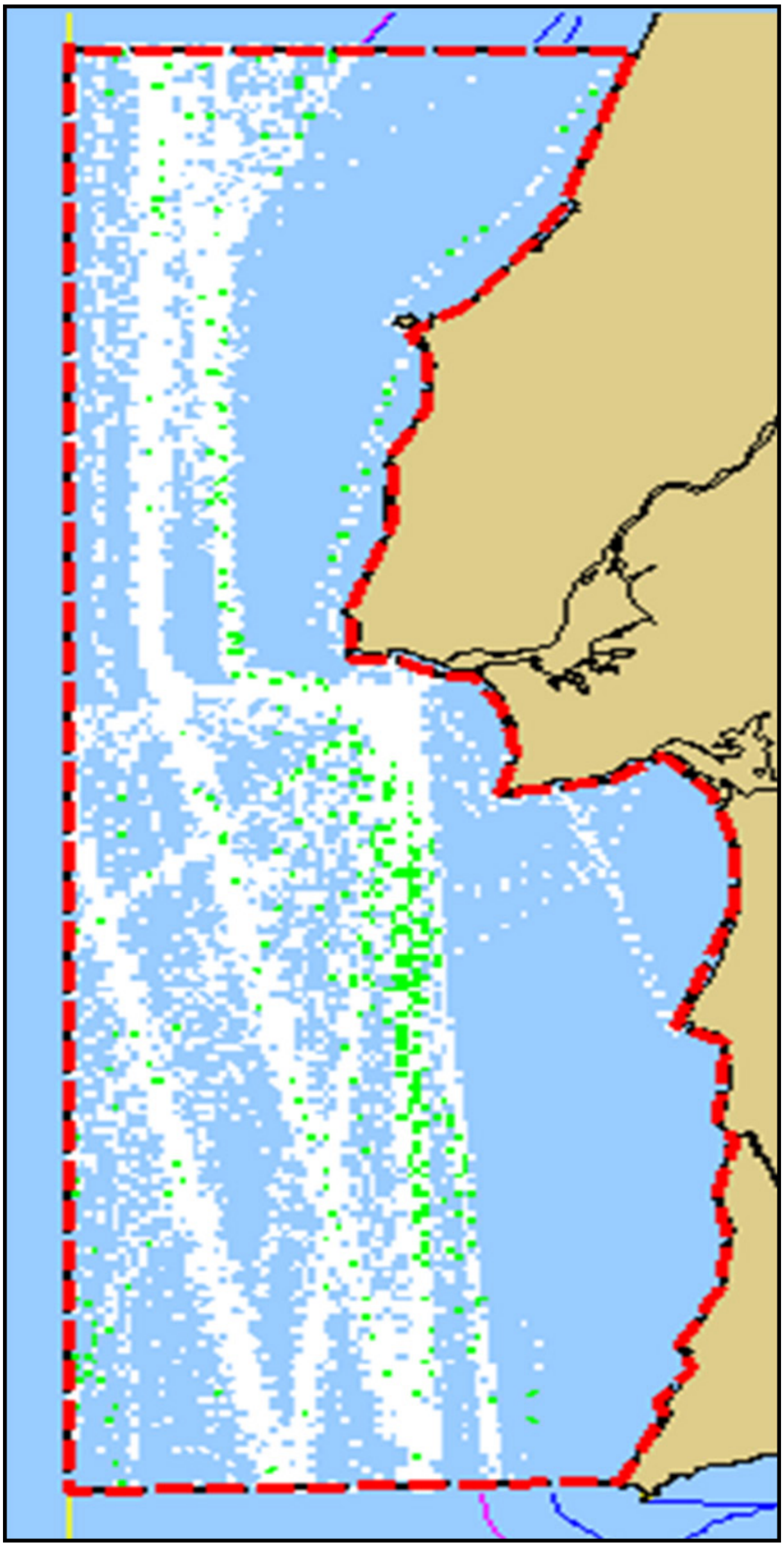

Figure 69. Polygon: “Comando de Zona Marítimo do Centro” . Mesh: 0.75 Nm^2

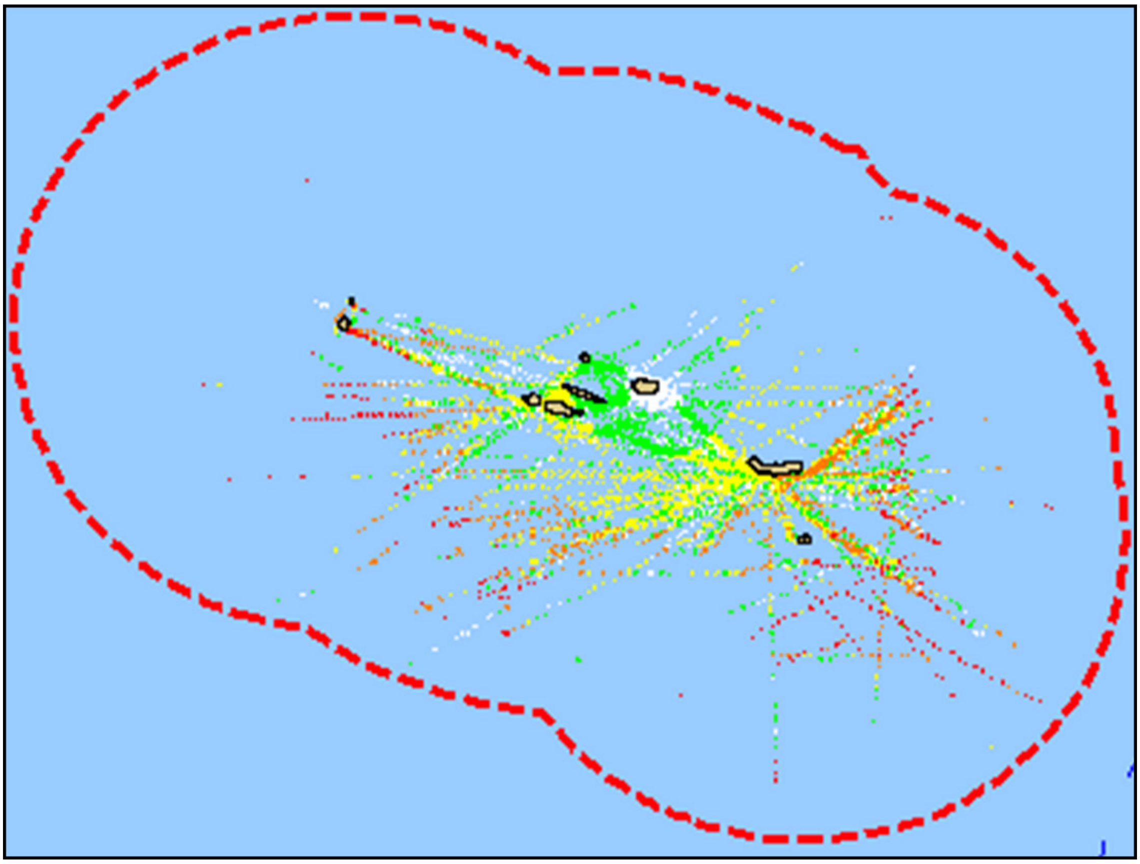

Figure 70. Polygon: “Açores”. Mesh: 2.5 Nm^2

Appendix A - Time performance for partial pilot method variants

The following tables show the time performance of the partial pilot method variants on the real cost sets of MMRO problems.

Table 27. Time performance of partial pilot method variations with one level and distance criteria

size			HC1d	HC1e	HC1p	HC2d	HC2e	HC2p
k	ns	np	t%	t%	t%	t%	t%	t%
	15	30	0.2 secs	0.2 secs	0.2 secs	0.6 secs	0.5 secs	0.5 secs
	20	32	0.3 secs	0.5 secs	0.4 secs	1.2 secs	1.1 secs	1.1 secs
3	30	24	1.0 secs	1.5 secs	1.1 secs	3.6 secs	3.5 secs	3.5 secs
	50	10	7.4 secs	12.0 secs	8.0 secs	19.4 secs	19.7 secs	19.6 secs
	80	5	49.0 secs	1 min, 22.9 secs	50.8 secs	2 mins, 9.1 secs	2 mins, 13.1 secs	2 mins, 0.7 secs
	15	30	0.7 secs	0.7 secs	0.7 secs	1.5 secs	1.4 secs	1.4 secs
	20	30	1.1 secs	1.3 secs	1.2 secs	2.9 secs	2.8 secs	2.8 secs
4	30	24	1.4 secs	1.9 secs	1.4 secs	4.7 secs	4.6 secs	4.6 secs
	50	10	8.8 secs	14.2 secs	9.7 secs	27.8 secs	28.7 secs	28.0 secs
	80	5	41.8 secs	1 min, 13.8 secs	46.8 secs	2 mins, 7.9 secs	2 mins, 9.7 secs	2 mins, 5.9 secs
	15	30	0.4 secs	0.5 secs	0.4 secs	1.1 secs	1.1 secs	1.2 secs
	20	30	0.6 secs	0.8 secs	0.6 secs	2.7 secs	2.7 secs	2.7 secs
5	30	24	1.7 secs	2.3 secs	1.7 secs	6.7 secs	6.8 secs	6.9 secs
	50	10	12.2 secs	18.5 secs	12.7 secs	39.7 secs	40.6 secs	40.6 secs
	80	5	1 min, 8.7 secs	2 mins, 3.5 secs	1 min, 16.4 secs	4 mins, 8.0 secs	4 mins, 12.3 secs	4 mins, 8.7 secs

Table 28. Time performance of partial pilot method variations with one level and ETA criteria

size			HC1d	HC1e	HC1p	HC2d	HC2e	HC2p
k	ns	np	t%	t%	t%	t%	t%	t%
	15	30	0.5 secs	0.5 secs	0.5 secs	0.9 secs	0.8 secs	0.8 secs
	20	32	0.9 secs	1.1 secs	1.0 secs	2.0 secs	2.0 secs	1.9 secs
3	30	24	1.9 secs	2.4 secs	2.0 secs	4.9 secs	4.8 secs	4.8 secs
	50	10	9.6 secs	14.6 secs	10.4 secs	24.6 secs	25.0 secs	24.2 secs
	80	5	33.7 secs	57.5 secs	34.6 secs	1 min, 21.6 secs	1 min, 24.5 secs	1 min, 17.7 secs
	15	30	0.3 secs	0.3 secs	0.3 secs	0.7 secs	0.7 secs	0.7 secs
	20	30	0.7 secs	0.9 secs	0.7 secs	1.9 secs	1.9 secs	1.9 secs
4	30	24	1.5 secs	2.0 secs	1.5 secs	4.9 secs	5.0 secs	5.0 secs
	50	10	9.5 secs	15.1 secs	10.6 secs	29.4 secs	30.1 secs	30.0 secs
	80	5	54.2 secs	1 min, 39.5 secs	1 min, 3.4 secs	2 mins, 48.6 secs	2 mins, 53.7 secs	2 mins, 57.7 secs
	15	30	0.3 secs	0.4 secs	0.3 secs	1.5 secs	1.4 secs	1.4 secs
	20	30	0.7 secs	0.9 secs	0.8 secs	3.2 secs	3.0 secs	3.1 secs
5	30	24	1.7 secs	2.5 secs	1.8 secs	8.3 secs	8.4 secs	8.5 secs
	50	10	15.0 secs	24.7 secs	16.7 secs	1 min, 2.3 secs	1 min, 1.7 secs	1 min, 1.9 secs
	80	5	1 min, 8.3 secs	2 mins, 4.3 secs	1 min, 16.4 secs	4 mins, 14.1 secs	4 mins, 22.8 secs	4 mins, 32.1 secs

Table 29. Time performance of partial pilot method with one level and profit criteria

size			HC1d	HC1e	HC1p	HC2d	HC2e	HC2p
k	ns	np	t%	t%	t%	t%	t%	t%
	15	30	0.2 secs	0.2 secs	0.2 secs	0.5 secs	0.5 secs	0.5 secs
	20	32	0.3 secs	0.4 secs	0.3 secs	0.9 secs	0.9 secs	0.9 secs
3	30	24	1.0 secs	1.4 secs	1.0 secs	2.8 secs	2.9 secs	2.8 secs
	50	10	6.4 secs	10.3 secs	7.1 secs	16.1 secs	16.3 secs	16.0 secs
	80	5	40.7 secs	1 min, 14.9 secs	42.7 secs	1 min, 36.7 secs	1 min, 45.0 secs	1 min, 36.2 secs
	15	30	0.3 secs	0.3 secs	0.3 secs	1.0 secs	0.9 secs	0.9 secs
	20	30	0.6 secs	0.8 secs	0.6 secs	2.2 secs	2.1 secs	2.1 secs
4	30	24	1.6 secs	2.2 secs	1.7 secs	6.7 secs	6.5 secs	6.5 secs
	50	10	10.1 secs	16.4 secs	11.6 secs	31.7 secs	32.7 secs	33.2 secs
	80	5	50.2 secs	1 min, 30.9 secs	57.1 secs	2 mins, 17.3 secs	2 mins, 22.5 secs	2 mins, 22.8 secs
	15	30	0.3 secs	0.4 secs	0.3 secs	1.3 secs	1.3 secs	1.3 secs
	20	30	0.7 secs	0.9 secs	0.7 secs	2.9 secs	2.9 secs	2.9 secs
5	30	24	1.8 secs	2.6 secs	1.9 secs	8.4 secs	8.5 secs	8.5 secs
	50	10	11.7 secs	19.5 secs	12.8 secs	44.0 secs	45.0 secs	44.9 secs
	80	5	1 min, 2.3 secs	1 min, 55.8 secs	1 min, 11.7 secs	3 mins, 34.3 secs	3 mins, 43.7 secs	3 mins, 39.2 secs

Table 30. Time performance of partial pilot method with two levels and distance criteria

size			HC1d	HC1e	HC1p	HC2d	HC2e	HC2p
k	ns	np	t%	t%	t%	t%	t%	t%
	15	30	0.8 secs	1.0 secs	0.8 secs	2.3 secs	2.3 secs	2.3 secs
	20	32	2.2 secs	2.9 secs	2.4 secs	6.5 secs	6.5 secs	6.4 secs
3	30	24	7.1 secs	9.8 secs	7.5 secs	20.8 secs	21.6 secs	21.4 secs
	50	10	49.0 secs	1 min, 17.5 secs	53.1 secs	2 mins, 3.4 secs	2 mins, 8.8 secs	2 mins, 7.6 secs
	80	5	4 mins, 47.0 secs	7 mins, 34.8 secs	5 mins, 4.2 secs	13 mins, 19.2 secs	14 mins, 34.6 secs	12 mins, 10.1 secs
	15	30	2.0 secs	2.4 secs	2.1 secs	6.7 secs	7.0 secs	6.8 secs
	20	30	4.6 secs	5.8 secs	4.7 secs	15.4 secs	15.9 secs	15.9 secs
4	30	24	13.1 secs	17.6 secs	13.2 secs	45.2 secs	46.9 secs	47.5 secs
	50	10	1 min, 25.1 secs	2 mins, 13.5 secs	1 min, 41.7 secs	4 mins, 26.9 secs	4 mins, 28.9 secs	4 mins, 18.2 secs
	80	5	7 mins, 25.2 secs	11 mins, 58.2 secs	7 mins, 36.6 secs	20 mins, 3.9 secs	20 mins, 4.9 secs	19 mins, 52.9 secs
	15	30	3.4 secs	4.2 secs	3.6 secs	13.4 secs	13.8 secs	13.6 secs
	20	30	7.1 secs	9.3 secs	7.4 secs	27.2 secs	28.3 secs	28.8 secs
5	30	24	17.6 secs	25.4 secs	18.7 secs	1 min, 17.2 secs	1 min, 17.5 secs	1 min, 18.4 secs
	50	10	2 mins, 2.8 secs	3 mins, 21.1 secs	2 mins, 10.6 secs	7 mins, 26.3 secs	7 mins, 46.8 secs	7 mins, 45.4 secs
	80	5	10 mins, 35.8 secs	18 mins, 56.1 secs	11 mins, 54.0 secs	36 mins, 48.0 secs	36 mins, 15.8 secs	38 mins, 25.2 secs

Table 31. Time performance of partial pilot method with two levels and ETA criteria

size			HC1d	HC1e	HC1p	HC2d	HC2e	HC2p
k	ns	np	t%	t%	t%	t%	t%	t%
	15	30	1.1 secs	1.3 secs	1.1 secs	2.9 secs	2.9 secs	2.9 secs
	20	32	2.4 secs	3.0 secs	2.6 secs	6.6 secs	6.8 secs	6.7 secs
3	30	24	7.2 secs	9.7 secs	7.3 secs	21.7 secs	21.9 secs	22.1 secs
	50	10	54.8 secs	1 min, 27.8 secs	59.0 secs	2 mins, 23.0 secs	2 mins, 22.8 secs	2 mins, 22.2 secs
	80	5	5 mins, 9.6 secs	8 mins, 37.8 secs	5 mins, 12.9 secs	12 mins, 29.4 secs	12 mins, 33.5 secs	11 mins, 20.9 secs
	15	30	2.7 secs	3.1 secs	2.8 secs	9.6 secs	9.4 secs	9.6 secs
	20	30	6.1 secs	7.8 secs	6.3 secs	22.8 secs	23.2 secs	23.0 secs
4	30	24	16.3 secs	23.2 secs	16.9 secs	1 min, 8.2 secs	1 min, 7.1 secs	1 min, 9.3 secs
	50	10	1 min, 28.4 secs	2 mins, 18.4 secs	1 min, 37.3 secs	4 mins, 26.3 secs	4 mins, 28.9 secs	4 mins, 26.0 secs
	80	5	7 mins, 34.0 secs	12 mins, 55.8 secs	8 mins, 34.9 secs	21 mins, 54.0 secs	21 mins, 51.2 secs	21 mins, 56.2 secs
	15	30	4.0 secs	4.7 secs	4.0 secs	16.8 secs	16.9 secs	16.7 secs
	20	30	7.8 secs	10.1 secs	8.0 secs	33.5 secs	33.8 secs	33.2 secs
5	30	24	17.7 secs	24.4 secs	18.1 secs	1 min, 17.4 secs	1 min, 17.5 secs	1 min, 18.2 secs
	50	10	2 mins, 2.7 secs	3 mins, 14.6 secs	2 mins, 9.9 secs	7 mins, 20.5 secs	7 mins, 29.7 secs	7 mins, 27.3 secs
	80	5	11 mins, 9.0 secs	19 mins, 28.9 secs	12 mins, 13.2 secs	37 mins, 37.1 secs	38 mins, 22.0 secs	37 mins, 50.7 secs

Table 32. Time performance of partial pilot method with two levels and profit criteria

size			HC1d	HC1e	HC1p	HC2d	HC2e	HC2p
k	ns	np	t%	t%	t%	t%	t%	t%
	15	30	0.7 secs	0.9 secs	0.7 secs	1.9 secs	1.9 secs	1.9 secs
	20	32	1.8 secs	2.2 secs	1.9 secs	4.5 secs	4.7 secs	4.5 secs
3	30	24	5.1 secs	7.2 secs	5.5 secs	14.9 secs	15.8 secs	15.1 secs
	50	10	35.5 secs	56.8 secs	38.7 secs	1 min, 26.9 secs	1 min, 27.5 secs	1 min, 26.6 secs
	80	5	4 mins, 47.3 secs	8 mins, 1.8 secs	3 mins, 36.4 secs	10 mins, 29.8 secs	9 mins, 43.8 secs	7 mins, 54.9 secs
	15	30	1.6 secs	1.9 secs	1.6 secs	5.1 secs	5.0 secs	5.0 secs
	20	30	3.6 secs	4.5 secs	3.8 secs	12.2 secs	12.1 secs	11.9 secs
4	30	24	9.9 secs	14.1 secs	10.3 secs	37.9 secs	38.9 secs	37.4 secs
	50	10	1 min, 2.0 secs	1 min, 38.0 secs	1 min, 7.6 secs	3 mins, 6.2 secs	3 mins, 13.7 secs	3 mins, 13.6 secs
	80	5	8 mins, 0.8 secs	14 mins, 33.0 secs	8 mins, 20.6 secs	23 mins, 16.7 secs	23 mins, 29.6 secs	19 mins, 28.0 secs
	15	30	2.6 secs	3.1 secs	2.7 secs	9.9 secs	10.2 secs	10.5 secs
	20	30	5.2 secs	6.9 secs	5.6 secs	19.8 secs	20.1 secs	20.4 secs
5	30	24	20.6 secs	29.0 secs	20.8 secs	1 min, 37.6 secs	1 min, 40.5 secs	1 min, 41.6 secs
	50	10	2 mins, 7.5 secs	3 mins, 32.7 secs	2 mins, 23.1 secs	8 mins, 4.0 secs	8 mins, 8.6 secs	8 mins, 12.0 secs
	80	5	11 mins, 55.2 secs	20 mins, 31.8 secs	11 mins, 31.7 secs	34 mins, 35.0 secs	40 mins, 43.2 secs	36 mins, 16.5 secs

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Appendix B - Parameters for *intlinprog* function

Intlinprog is a Mixed-integer linear programming (MILP) solver available in MATLAB's optimization toolbox. This solver finds the minimum of a problem specified by

$$\min f^T x \text{ subject to } \begin{cases} x(intcon)\ are\ integers \\ A \cdot x \leq b \\ Aeq \cdot x = beq \\ lb \leq x \leq ub \end{cases}$$

Where $f, x, intcon, b, beq, lb,$ and ub are vectors, and A and Aeq are matrices. Intlinprog uses a basic strategy involving six consecutive stages (see [224] for a detailed description) to solve mixed-integer linear programs. These stages are:

1. Reduce the problem size using Linear Program Preprocessing.
2. Solve an initial relaxed (noninteger) problem using Linear Programming.
3. Perform Mixed-Integer Program Preprocessing to tighten the LP relaxation of the mixed-integer problem.
4. Try Cut Generation to further tighten the LP relaxation of the mixed-integer problem.
5. Try to find integer-feasible solutions using heuristics.
6. Use a Branch and Bound algorithm to search systematically for the optimal solution. This algorithm solves LP relaxations with restricted ranges of possible values of the integer variables. It attempts to generate a sequence of updated bounds on the optimal objective function value.

Intlinprog function can be called with the following syntax:

x = intlinprog(f,intcon,A,b,Aeq,beq,lb,ub,options)

The options input is a MATLAB's structure with several fields that allow the user to set specific options for the intlinprog function. The options structure can be created by calling the optimoptions function with the following sintax:

options=optimoptions(@intlinprog)

Several runs were performed with different settings in the options structure to evaluate the best choice of parameters presented in Section 3.6. From this experiments a final set of parameters were chosen to solve the MMRO instances. The computational results were obtained with the following parameters:

MaxTime=500000;
RootLPMaxIter = 30000000;
LPMaxIter=1000000000;
Heuristics='none';
BranchingRule='mostfractional';

The MaxTime parameter is a positive real number that is the maximum time in seconds that intlinprog runs. For our problems the maximum time was set to 500000 seconds or aproximatly 5 days and 18 hours. LPMaxIter is the maximum number of simplex algorithm iterations per node during the branch-and-bound process. RootLPMaxIter is a nonnegative integer that is the maximum number of simplex algorithm iterations to solve the initial linear programming problem. Stage 5 was not performed while solving the MMRO instances.